Predicate logic

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Synonyms. First-order logic.

Question 1. Describe this discipline/sub-discipline, and some of its more recent developments.

Predicate logic is a sub-discipline of logic that had its roots in the last quarter of the nineteenth century, though it had to wait until the second decade of the twentieth century for a solid foundation. Like any other logic, it is concerned with the validity of arguments, though not of any kind: its interest lies in reasoning about what is universally true. As such, predicate logic is especially suited to reason about mathematical statements and can be considered a generalization of Aristotelian *syllogisms*. Predicate logic goes beyond syllogisms by introducing predicates with arbitrary numbers of arguments, and quantifiers that allow to refer either to all or to some of the elements in the universe that is under consideration. It has a proof theory, which consists of a set of rules that describe how to mechanically derive sentences from a given set of premises (and such derivations are called "proofs"), as well as a model theory that assigns meaning to the sentences with respect to *structures* so that a given sentence is either true or false in a given structure. Predicate logic is sound, meaning that every sentence that can be derived using the rules in its proof theory holds in every possible structure. More importantly, a fundamental result by Gödel in 1929 shows that predicate logic is also complete in the sense that if a sentence is true in every structure then it can be derived from the empty set of premises.

Question 2

(a) To what extent does this discipline/sub-discipline self-identify as a science? How so? In what way, or why not at all?

Logic, and in particular predicate logic, is a formal science. As opposed to natural sciences, knowledge in predicate logic is not obtained through observations and *empirical study* but from the analysis and processing of certain symbols and rules. It is also formal in the sense that it provides a framework in which reasoning can take place, the form of the argumentation being the only element that determines whether it is valid or not (disregarding any purported meaning).

(b) To what extent does this discipline/sub-discipline self-identify as a religion? How so? In what way, or why not at all?

This discipline does not identify itself as a religion. It lacks a liturgy, rites, divine entities, a moral, or any of the elements pertaining to a religion.

Question 3. What makes this discipline/sub-discipline distinctive among the other disciplines/sub-disciplines?

Predicate logic, with its focus on universally true statements, has proved to be sufficient in principle to formalize and reason about the main bulk of mathematics. In addition, predicate logic satisfies many good properties (such as completeness), that not all variants of logic posses, and it has been shown in a precise manner that a logic more *expressive* than predicate logic must give up some of these properties in exchange of its additional power. However, different disciplines (like computer science) require other kinds of reasoning, such as probabilistic, subjunctive, temporal, or fuzzy, to name a few. Dealing with that kind of arguments in predicate logic is convoluted at best, while other branches of logic are more appropriate and should be used instead: probabilistic logic, temporal logic, fuzzy logic, ...

Question 4. To what extent does this discipline/sub-discipline see itself as relevant to, interested in the scholarly area called 'Science and Religion'? If interested, in what way? If not interested, why not?

Predicate logic is not interested in the area 'Science and Religion'. It is relevant to it, though, in the same broad sense that logic in general is: it is a useful tool which provides a universal language with which to reason about any subject matter.

Question 5. What are the sources of authority for this discipline/sub-discipline? What makes these sources authoritative?

The sources of authority in predicate logic are the peer-reviewed papers published in journals and conferences, where new results are proved in a rigorous mathematical manner. Among the many prominent logicians, Kurt Gödel (1906–1978) and Alfred Tarski (1901–1983) can be singled out as two of the most influential and important of all time: neither created this sub-discipline, but they both changed it profoundly and set forth research in new and unexpected directions.

Question 6. What are the ethical principles that guide this discipline/sub-discipline?

Since predicate logic has no direct effect and is not affected by the physical world, ethical or moral considerations have no especial application here. Being honest, as in any scientific discipline, would be the main guiding principle.

Question 7. What are the key values of this discipline/sub-discipline?

Providing an abstract framework in which to reason in a precise manner about any subject and, especially, about mathematics, for which it offers a way of attacking many interesting problems. It is a potent tool to avoid ambiguities when posing an argument as well as when solving it.

Question 8. How does this discipline/sub-discipline define/conceptualize the following?

- **Nature / world**

  The empirical world is of no concern to predicate logic due to its formal character. Thus, there is no explicit treatment of these terms within this sub-discipline.

- **Human being**

  Again, this concept is not part of the field of study of predicate logic and thus the human being is not defined within this sub-discipline.

- **Life (and origins of) / death**

  By the same reason, questions about life and death do not apply to this sub-discipline.

- **Reality**

  As a formal science predicate logic is not directly concerned with reality. At most, it can be said that predicate logic's principles of inference are not arbitrary but inspired on "what happens in reality", as in the case of the principle of identity (A is A) or non-contradiction (it cannot be the case that both A and not A).

- **Knowledge**

  Reasoning about knowledge is more suited for other sub-disciplines of logic such as modal logic. Though not concerned with knowledge, the renowned Gödel's (first) incompleteness theorem has been widely put to philosophical uses with regard the limitations of what can be known. Roughly, this theorem states that given a set of *axioms* for a sufficiently strong (mathematical) system, there is always a sentence such that neither it nor its negation can be derived from the axioms using the proof theory; this implies that there are truths in the system for which no proof can be found.

- **Truth**

  Truth as an absolute notion was abandoned by logicians during the first thirty years of the twentieth century. A predicate logic sentence on its own is, in general, neither true nor false, but requires a context, a given structure with respect to which it does have a truth value. Moreover, the completeness of predicate logic allows to deal with this semantic notion in a mechanical way: logically valid sentences, those which are true with respect to every structure, are exactly those that can be derived by means of the rules in the proof theory. (Different logics may use different classes of structures, so that logically valid sentences may vary from

one to another.) Note that this is in sharp contrast with truth in concrete mathematical systems where, by the incompleteness theorem, there are true sentences which are not provable.

- **Perception**

  Predicate logic is not concerned with perception since it does not have to deal with sensorial experiences.

- **Time**

  Time is not treated explicitly within this sub-discipline. Even though time could be formalized so as to reason about facts that take it into account, there are many variants of temporal logic more suited for this purpose.

- **Consciousness**

  Consciousness is not treated within this sub-discipline.

- **Rationality / reason**

  Predicate logic does not define what reason is nor treats it. However, as a formal science it is driven by rationality alone, with complete disregard of the physical world.

- **Mystery**

  Mystery has no place in this sub-discipline. On the contrary, predicate logic has played an enlightening role in the clarification of the nature of many semantic paradoxes in the natural language through their rephrasing as logical sentences.

What (additional) issues/themes/concepts are especially relevant for this discipline/sub-discipline as regards Science and Religion engagement? In what way are these issues/themes/concepts critical?

A critical issue, already sketched in the answers to the previous questions, is the notion of proof and the distinction between true and provable sentences. In particular, the results by Gödel and Tarski show that the notion of provability in a given system is definable within the system itself (assuming minimum requirements about its expressive power), whereas the corresponding notion of truth can only be defined in another system that adequately extends the original.

Glossary

- *Axiom.* An axiom is simply a sentence in the logic which, in combination with others, is used as a premise to derive new sentences. Intuitively, it expresses an evident proposition about some concrete system that is taken for granted when reasoning about it.

- *Empirical study.* Study based on experience and the observation of natural phenomena.

- *Expressive.* A logic L is said to be at least as expressive as another logic L' if, for every sentence in L', there exists a sentence in L such that the structures that make the first sentence true are exactly the same as the structures that make the second true.

- *Structure.* A structure in predicate logic is a model in which to interpret sentences. It consists of a universe of elements, together with operations over these elements associated to the symbols in the language.

- *Syllogism.* A logical argument which reaches a conclusion from two given premises of a certain form. Syllogisms were classified by Aristotle, who identified those that correspond to valid inferences.

References

D. van Dalen. *Logic and Structure. Fourth Edition.* Springer, 2004.

H.-D. Ebbinghaus, J. Flum, and W. Thomas. *Mathematical Logic. Second Edition.* Springer, 1996.

H. B. Enderton. *A Mathematical Introduction to Logic, Second Edition.* Academic Press 2000.

D. M. Gabbay and F. Guenthner, editors. *Handbook of Philosophical Logic. 2nd edition.* Springer. In eighteen volumes, the first published in 2001.

K. Gödel. *Collected Works.* Oxford University Press. In five volumes, the first published in 1986.

S. G. Simpson. Logic and Mathematics. In S. Rosen (ed.) *The Examined Life: Readings from Western Philosophers from Plato to Kant.* Random House Reference, 2000.

A. Tarski. *Logic, Semantics, Metamathematics, papers from 1923 to 1938.* Hackett Publishing Company, 1983.