Symbolic logic

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Synonyms. Mathematical logic; formal logic; modern logic.

Question 1. Describe this discipline/sub-discipline, and some of its more recent developments.

The roots of symbolic logic can be traced as far back as to the study of *syllogisms* by Aristotle. However, it was not until the second half of the nineteenth century that its modern form began crystallizing, whereas solid foundations had to wait until the second decade of the twentieth century.

Symbolic logic is concerned with the validity of arguments, for whose study it abstracts away from concrete instances: the validity of an argument depends exclusively on the form of the premises and the conclusion, not on whether these are true or false. For example, if "Anne and Bob are not both clorophastreo" and "Bob is clorophastreo", then it follows that "Anne is clorophastreo" regardless of the meaning of the adjective "clorophastreo". Symbolically, concluding "A" from the premises "not (A and B)" and "not B" is a valid inference rule.

Among the different logics (sub-disciplines) encompassed in this field, there is one that deserves to be singled out for its relevance: predicate logic. Its interest lies in reasoning about what is universally true and, as such, it is especially suited to reason about mathematical statements and can be considered a generalization of Aristotelian syllogisms. Predicate logic goes beyond syllogisms by introducing predicates with arbitrary numbers of arguments, and quantifiers that allow to refer either to all or to some of the elements in the universe that is under consideration. It has a proof theory, which consists of a set of rules that describe how to mechanically derive sentences from a given set of premises (and such derivations are called "proofs"), as well as a model theory that assigns meaning to the sentences with respect to *structures* so that a given sentence is either true or false in a given structure. Predicate logic is sound, meaning that every sentence that can be derived using the rules in its proof theory holds in every possible structure. More importantly, a fundamental result by Gödel in 1929 shows that predicate logic is also complete in the sense that if a sentence is true in every structure then it can be derived from the empty set of premises.

But this is not the only possibility. Within the discipline of symbolic logic, a veritable cornucopia of different logics has sprung during the last century. Although there is no

rigorous, widely agreed on definition of what constitutes a logic, they share the following features already mentioned for predicate logic:

- A formal language over which sentences, representing statements, can be built.
- A proof theory.
- A model theory.

Actually, it is not necessary for a logic to have both a proof and a model theory, and it is quite possible that one of them is emphasized over the other; also, depending on the relationship between them, the corresponding logic will be sound and/or complete.

We can deviate from predicate logic by making suitable choices among several (independent) design decisions that can be taken when defining a new logic. To cite some of the most important:

- Quantifiers. It is possible (as in predicate logic) to have quantifiers to refer to all or to some of the elements in a structure, but also not to have quantifiers at all (as in propositional logic), or quantifiers to range over sets of elements (second-order logic). Other alternatives include referring to almost all of the elements, only to a few of them, or to infinitely many.
- Intuitionism. In traditional mathematics, non-constructive arguments are valid. In particular, the law of excluded middle "A or not A" is always true and this is reflected in the inference rules of logics such as predicate logic. Intuitionism objects to non-constructivism and in the logics that follow this school of thought such inferences are not valid and thus are ruled out.
- Infinitary logic. Sentences are finite sequences of symbols built out of constructors, such as disjunction, that usually take a finite number of arguments. However, it is also possible to design logics with infinitary constructors to represent infinitely long formulas.
- Many-valued logic. In classical logics there are only two truth values, true and false; however, some logics extend this set to account for many more possibilities in an attempt to model concepts such as uncertainty or vagueness.

Question 2

(a) To what extent does this discipline/sub-discipline self-identify as a science? How so? In what way, or why not at all?

Symbolic logic is a formal science. As opposed to natural sciences, knowledge in symbolic logic is not obtained through observations and *empirical study* but from the analysis and processing of certain symbols and rules. It is also formal in the sense that it provides a framework in which reasoning can take place, the form of the argumentation being the only element that determines whether it is valid or not (disregarding any purported meaning).

(b) To what extent does this discipline/sub-discipline self-identify as a religion? How so? In what way, or why not at all?

This discipline does not identify itself as a religion. It lacks a liturgy, rites, divine entities, a moral, or any of the elements pertaining to a religion.

Question 3. What makes this discipline/sub-discipline distinctive among the other disciplines/sub-disciplines?

As opposed to symbolic logic, informal logic aims at studying the validity of arguments without formalizing them. In fact, it has more to do with the social practice of argumentation as it occurs in everyday life and with skills such as critical perspective, than with symbolic logic.

As compared to other mathematical branches such as analysis or geometry, research on symbolic logic has been motivated by the study of the foundations of mathematics. Its actual use in practice has been rather limited and confined to establishing which parts of mathematics can be formalized in which logics. By contrast, reasoning of a probabilistic, subjunctive, temporal, or fuzzy kind is required for applications in computer science, and symbolic logic has readily provided it.

Question 4. To what extent does this discipline/sub-discipline see itself as relevant to, interested in the scholarly area called 'Science and Religion'? If interested, in what way? If not interested, why not?

Symbolic logic is not interested in the area 'Science and Religion'; its field of study abstracts from the real world and focuses instead on mental constructions. It is relevant to it, though, in the same broad sense that logic in general is: it is a useful tool which provides a universal language with which to reason about any subject matter.

The above notwithstanding, symbolic logic has been used by some authors (Gödel among them) to formalize ontological arguments about the existence and non-existence of God.

Question 5. What are the sources of authority for this discipline/sub-discipline? What makes these sources authoritative?

The sources of authority in symbolic logic are the peer-reviewed papers published in journals and conferences, where new results are proved in a rigorous mathematical manner. Among the many prominent logicians, Kurt Gödel (1906–1978) and Alfred Tarski (1901–1983) can be singled out as two of the most influential and important of all time: neither created this discipline, but they both changed it profoundly and set forth research in new and unexpected directions.

Question 6. What are the ethical principles that guide this discipline/sub-discipline?

Since logic has no direct effect and is not affected by the physical world, ethical or moral considerations have no especial application here. Being honest, as in any scientific discipline, would be the main guiding principle.

Question 7. What are the key values of this discipline/sub-discipline?

Providing an abstract framework in which to reason in a precise manner about many subjects, especially in mathematics and computer science, for which it offers a way of attacking many interesting problems. It is a potent tool to avoid ambiguities when posing an argument as well as when solving it.

Question 8. How does this discipline/sub-discipline define/conceptualize the following?

- **Nature / world**

  The empirical world is of no concern to symbolic logic due to its formal character. Thus, there is no explicit treatment of these terms within this discipline.

- **Human being**

  Again, this concept is not part of the field of study of symbolic logic and thus the human being is not defined within this discipline.

- **Life (and origins of) / death**

  By the same reason, questions about life and death do not apply to this discipline.

- **Reality**

  As a formal science, symbolic logic is not directly concerned with reality. At most, it can be said that the principles of inference of its different sub-disciplines are not arbitrary but inspired on what happens in reality, as in the case of the principle of identity (A is A) or non-contradiction (it cannot be the case that both A and not A) for predicate logic. Note, however, that different logics may have different, even contradictory principles; for example, the law of excluded middle does not hold for intuitionistic logics.

- **Knowledge**

  Among the many variants of symbolic logic, modal logic introduces connectives to deal with modalities such as necessity, possibility, or provability. In particular, the modal logics of knowledge are concerned with modeling and reasoning about the knowledge that a set of entities (humans, computers, ...) hold over the world and about each other's knowledge.

  In a different direction, it is also worth mentioning that the renowned Gödel's (first) incompleteness theorem for predicate logic has been widely put to philosophical uses with regard to the limitations of what can be known. Roughly, this theorem states that given a set of *axioms* for a sufficiently strong (mathematical) system, there is always a sentence such that neither it nor its negation can be derived from the axioms using the proof theory; this implies that there are truths in the system for which no proof can be found.

- **Truth**

  Truth as an absolute notion was abandoned by logicians during the first thirty years of the twentieth century. A symbolic logic sentence on its own is, in general,

neither true nor false (nor anything in between), but requires a context, a given structure with respect to which it does have a truth value. In the particular case of predicate logic, its completeness allows to deal with this semantic notion in a mechanical way: logically valid sentences, those which are true with respect to every structure, are exactly those that can be derived by means of the rules in the proof theory. (Different logics may use different classes of structures, so that logically valid sentences may vary from one to another.) Note that this is in sharp contrast with truth in concrete mathematical systems where, by the incompleteness theorem, there are true sentences which are not provable.

- **Perception**

  Symbolic logic is not concerned with perception since it does not have to deal with sensorial experiences.

- **Time**

  Temporal logic is a kind of modal logic especially designed to reason about time. There are actually many variants, catering for the future, for both the future and the past, for all possible futures, for a single future at a time, ...

- **Consciousness**

  Consciousness is not treated within this sub-discipline.

- **Rationality / reason**

  Symbolic logic does not define what reason is nor treats it. However, as a formal science it is driven by rationality alone, with complete disregard of the physical world.

- **Mystery**

  Mystery has no place in this discipline. On the contrary, symbolic logic has played an enlightening role in the clarification of the nature of many semantic paradoxes in the natural language through their rephrasing as logical sentences.

What (additional) issues/themes/concepts are especially relevant for this discipline/sub-discipline as regards Science and Religion engagement? In what way are these issues/themes/concepts critical?

A critical issue, already hinted at in the answers to the previous questions, is the notion of proof and the distinction between true and provable sentences. In particular, the results by Gödel and Tarski show that the notion of provability in a given system is definable within the system itself (assuming minimum requirements about its *expressive* power), whereas the corresponding notion of truth can only be defined in another system that adequately extends the original.

Glossary

- *Axiom.* An axiom is a sentence in the logic which, in combination with others, is used as a premise to derive new sentences. Intuitively, it expresses an evident

proposition about some concrete system that is taken for granted when reasoning about it.

- *Empirical study.* Study based on experience and the observation of natural phenomena.

- *Expressive.* A logic L is said to be at least as expressive as another logic L' if, for every sentence in L', there exists a sentence in L such that the structures that make the first sentence true are exactly the same as the structures that make the second true.

- *Structure.* A structure in symbolic logic is a model, a world in which to interpret sentences. Its form depends on the concrete logic but typically consists of a universe of elements, together with operations over these elements associated to the symbols in the language.

- *Syllogism.* A logical argument which reaches a conclusion from two given premises of a certain form. Syllogisms were classified by Aristotle, who identified those that correspond to valid inferences.

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