On the Unification of Process Semantics: Logical Semantics

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Abstract. We continue with the task of obtaining a unifying view of process semantics by considering in this case the logical characterization of the semantics. Recently, a unified presentation of both the observational and the equational characterizations of the semantics has been obtained. As in those papers, we start by considering the classical Linear time-branching time spectrum developed by R.J. van Glabbeek. He provided indeed a logic characterization of most of the semantics classified in his spectrum but, as was the case for the other kinds of semantics, it lacks uniformity. In this paper, we provide a uniform logical characterization of all the semantics in the extended spectrum. The common structure of the formulas that constitute all the corresponding logics gives us a much clearer picture of the spectrum, clarifying the relations between the different semantics, and allows us to develop generic proofs of some general properties of the semantics.

1 Introduction

The definition of the semantics for concurrent / non-deterministic processes is a delicate question. As soon as the effect of non-determinism is taken into account we have to decide to which extent we will do so. Trace semantics which were adequate for deterministic systems obviously do not consider non-determinism at all. Instead, bisimulation semantics captures all the information induced by the choices at the observed process. There are different semantics for processes in the literature. The most popular of them were collected in Van Glabbeek's linear time-branching time spectrum [vG01], after being introduced along the years by different authors. But it is not only at the level of discrimination where we have a choice for defining these semantics, we can also choose between different frameworks to describe the different semantics, so we have operational, observational, testing, logical and equational semantics.

In [vG01] we find the famous picture of the ltbt-spectrum (Figure 1) and descriptions of all the semantics in it including observational / testing, logical and equational (when possible) characterizations for each of the presented semantics. Certainly, the basic elements used in the characterizations for a given framework are related, but a more systematic approach is desirable. In [dFGP09a,dFGP09b], it has been developed a unified presentation of both the

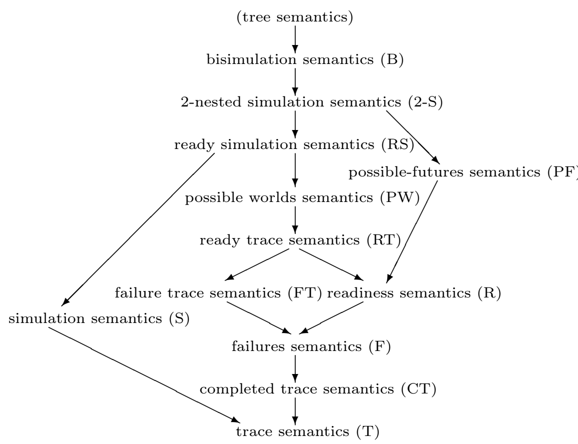

Fig. 1. ltbt-spectrum

observational and the equational semantics, which shows how the generic definitions allow to relate both without repeating similar arguments.

In this paper we present a unified view of the logical semantics by showing how different subsets of the Hennessy-Milner logic HML [HM85] characterize each of the semantics in the spectrum. Certainly, the logical characterizations presented in [vG01] were also subsets of HML, however in that paper the author looked for sets of formulas as small as possible, probably driven by the idea that a smaller set of formulas makes any study based on it simpler. Instead, we will follow here the opposite approach, so that we are able to obtain more natural characterizations which immediately show the relations between the different semantics.

As already happened in [dFGP09a,dFGP09b], our unified logical semantics will provide an enlarged spectrum (Figure 2) with a clearer structure and additional nodes which correspond to new semantics that in some cases have been also defined using different frameworks by several authors. In particular, we will show the logical characterization of revivals semantics introduced by Bill Roscoe in [Ros09], that was already axiomatized in [dFGP09a].

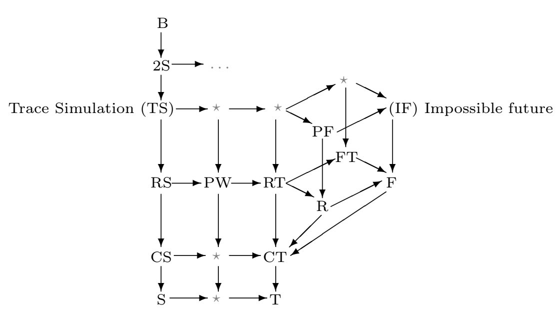

Fig. 2. (A part of) the new spectrum

Moreover, we "discover" here the semantics of minimal readies that was not included in the previous version of the extended spectrum, because the development of the observational and equational frameworks did not detect their existence, while now in the logical framework its definition comes quite naturally. Finally, by considering the logical characterizations we have been able to detect some new basic facts of the elements in the hierarchy of semantics, discovering more new semantics that fill some gaps in it. We have, also been able to discover a minor mistake in the classical logical characterization of one of the semantics: possible worlds, that has been easily corrected when applying our uniform characterization.

2 Preliminaries

We will not repeat here the long list of original definitions of all the semantics in van Glabbeek's spectrum. Please, look at [vG01]. The systematic classification of all these semantics using both observational and equational characterizations can be found at [dFGP09a,dFGP09b]. The main ingredient in this classification, that of course was already present at the original spectrum, is the distinction between branching and linear time semantics. In particular, all the pure branching semantics can be described by means of N-constrained simulations as defined at [dFEGR08].

Definition 1 ([dFEGR08]). *Given a relation N over BCCSP process, an N-constrained simulation is a relation S_N such that pS_Nq implies: $S_N \subseteq N$ and for every $a \in Act$, if $p \xrightarrow{a} p'$ there exists q', $q \xrightarrow{a} q'$ and $p'S_Nq'$. We say that process p is N-simulated by process q, or that q N-simulates p, written $p \sqsubseteq_{NS} q$, whenever there exists an N-constrained simulation S_N such that pS_Nq.*

2.1 Van Glabbeek's logical characterizations for process semantics

Van Glabbeek also presented in [vG01] a logical characterization of the semantics in the (classical) linear time-branching time spectrum. These logics are sublanguages of the Hennessy-Milner logic [HM85], $\mathcal{L}_{HM}$, characterizing the bisimulation semantics in the general (possibly infinitary) case.

Definition 2 (Hennessy-Milner logic, HML). *The set $\mathcal{L}_{HM}$ of Hennessy-Milner logical formulas is defined by: $\top \in \mathcal{L}_{HM}$; if φ, $\varphi_i \in \mathcal{L}_{HM}$ $\forall i \in I$ and $a \in Act$ then we have $\bigwedge_{i\in I} \varphi_i$, $a\varphi$, $\neg\varphi \in \mathcal{L}_{HM}$.*

For each labelled transition system $\mathbb{P}$, the satisfaction relation $\models \subseteq \mathbb{P} \times \mathcal{L}_{HM}$ is defined by:

- $p \models \top$ *for all* $p \in \mathbb{P}$;
- $p \models a\varphi$ *if there exists* $q \in \mathbb{P} : p \xrightarrow{a} q$ *and* $q \models \varphi$;
- $p \models \bigwedge_{i\in I} \varphi_i$ *if for all* $i \in I : p \models \varphi_i$.
- $p \models \neg\varphi$ *if* $p \not\models \varphi$.

The finite version of this logic $\mathcal{L}_{HM}^f$ uses binary conjunction $\wedge$ instead of the general (possibly infinite) conjunction $\bigwedge_{i\in I}$. The former can be obtained as a particular case of this general conjunction operator: $\varphi \wedge \psi$, is obtained taking $I = \{1,2\}$, $\varphi_1 = \varphi$, $\varphi_2 = \psi$. In this way we obtain $\mathcal{L}_{HM}^f \subseteq \mathcal{L}_{HM}$, and we will say that $\mathcal{L}_{HM}^f$ is the *finite part* of the whole logic $\mathcal{L}_{HM}$. It is well known that $\mathcal{L}_{HM}^f$ characterizes the bisimulation semantics between finite image processes, that are those that do not allow infinitely branching for any action $a \in Act$ at any state.

Van Glabbeek uses $\mathcal{L}_B$ to refer to $\mathcal{L}_{HM}$ in [vG01].

Definition 3. *Any subset $\mathcal{L}$ of $\mathcal{L}_{HM}$ induces a logical semantics for processes, given by the preorder $\sqsubseteq_{\mathcal{L}}$: We have $p \sqsubseteq_{\mathcal{L}} q$ if, and only if, for all $\varphi \in \mathcal{L}$ we have $p \models \varphi \Rightarrow q \models \varphi$. We say that $\mathcal{L}$ and $\mathcal{L}'$ are equivalent, and we write $\mathcal{L} \sim \mathcal{L}'$, if they induce the same semantics, that is $\sqsubseteq_{\mathcal{L}} = \sqsubseteq_{\mathcal{L}'}$.*

Table 1 contains the logical characterization of each of the semantics in van Glabbeek's spectrum: $\mathcal{L}_Z$ denotes each of the logical languages, the dots indicate the clauses that we need to introduce to obtain the corresponding languages, and the boxes marked with ν correspond to rules that are also valid in $\mathcal{L}_Z$, but not needed. The following connectives, which appear in the table, are not in $\mathcal{L}_{HM}$, but can be obtained as syntactic sugar, as follows:

$$\top := \bigwedge_{i\in\emptyset} \varphi_i \qquad \widetilde{X} := \bigwedge_{a\in X} \neg a\top \qquad \widetilde{X}\varphi' := \widetilde{X} \wedge \varphi' \qquad 0 := \widetilde{Act}$$

$$\varphi_1 \wedge \varphi_2 := \bigwedge_{i\in\{1,2\}} \varphi_i \qquad X := \bigwedge_{a\in X} a\top \wedge \bigwedge_{a\notin X} \neg a\top \qquad X\varphi' := X \wedge \varphi' \qquad \widetilde{a} := \neg a\top$$

Disjunction does not appear in $\mathcal{L}_{HM}$, and therefore neither in any of the logics $\mathcal{L}_Z$ characterizing the semantics in the linear time-branching time spectrum. Probably, it is folklore that it can be added in all cases without changing the expressive power of each of these logics, but since we have not found a clear statement in this direction in any of our references, next we establish the result and comment on its proof.

Proposition 1 *All the logics considered above are semantically closed under disjunction, that is, if we define $\mathcal{L}_Z^{\vee}$ with $Z \in \{T, CT, F, FT, R, RT, PF, S, CS, RS, 2S, PW, B\}$, by adding the clause $\sigma_i \in \mathcal{L}_Z^{\vee}\ \forall i \in I \Rightarrow \bigvee_{i\in I} \sigma_i \in \mathcal{L}_Z^{\vee}$ to the clauses which define each semantics $\mathcal{L}_Z$, replacing $\mathcal{L}_Z$ by $\mathcal{L}_Z^{\vee}$ at each of the other clauses, and making $p \models \bigvee \sigma_i$ iff $\exists i \in I$: $p \models \varphi_i$, we will get an equivalent logic.*

Proof. It is interesting to observe that even if the result is valid for all the semantics, the reason behind, is not the same as in the case of bisimulation. In that case, we only need to apply the De Morgan laws to get the "definition" of $\vee$ as a combination of $\neg$ and $\wedge$. However, for the rest of the semantics, we do not have negation as "constructor", but $\vee$ distributes over $\wedge$ and the prefix operator (because $\bigvee a\varphi_i = a \bigvee \varphi_i$), while negation is never applied to a formula $\varphi' \in \mathcal{L}_Z^{\vee}$.

Formulas \ Semantics ($\mathcal{Z}$)	T	S	CT	CS	F	FT	R	RT	PW	RS	PF	2S	B
$\top \in \mathcal{L}_{\mathcal{Z}}$	•	ν	•	ν	•	•	•	•	ν	ν	ν	ν	ν
$\mathbf{0} \in \mathcal{L}_{\mathcal{Z}}$			•	•	ν	ν	ν	ν	ν	ν	ν	ν	ν
$\varphi \in \mathcal{L}_{\mathcal{Z}}$, $a \in Act \Rightarrow$ $a\varphi \in \mathcal{L}_{\mathcal{Z}}$	•	•	•	•	•	•	•	•	ν	•	•	•	•
$X \subseteq Act \Rightarrow$ $\widetilde{X} \in \mathcal{L}_{\mathcal{Z}}$					•	ν	ν	ν	ν	ν	ν	ν	ν
$X \subseteq Act \Rightarrow$ $X \in \mathcal{L}_{\mathcal{Z}}$							•	ν	•	•	ν	ν	ν
$\varphi \in \mathcal{L}_{\mathcal{Z}}$, $X \subseteq Act \Rightarrow$ $\widetilde{X}\varphi \in \mathcal{L}_{\mathcal{Z}}$						•		ν	ν	ν		ν	ν
$\varphi \in \mathcal{L}_{\mathcal{Z}}$, $X \subseteq Act \Rightarrow$ $X\varphi \in \mathcal{L}_{\mathcal{Z}}$								•	ν	ν		ν	ν
$\varphi_i \in \mathcal{L}_{\mathcal{Z}}\ \forall i \in I \Rightarrow$ $\bigwedge_{i\in I} \varphi_i \in \mathcal{L}_{\mathcal{Z}}$		•		•						•		•	•
$X \subseteq Act$, $\varphi_a \in \mathcal{L}_{PW}\ \forall a \in X \Rightarrow$ $\bigwedge_{a\in X} a\varphi_a \in \mathcal{L}_{\mathcal{Z}}$									•	ν		ν	ν
$\varphi_i, \varphi_j \in \mathcal{L}_T\ \forall i \in I\, \forall j \in J \Rightarrow$ $\bigwedge_{i\in I} \varphi_i \wedge \bigwedge_{j\in J} \neg\varphi_j \in \mathcal{L}_{\mathcal{Z}}$											•	ν	ν
$\varphi \in \mathcal{L}_S \Rightarrow$ $\neg\varphi \in \mathcal{L}_{\mathcal{Z}}$												•	ν
$\varphi \in \mathcal{L}_{\mathcal{Z}} \Rightarrow$ $\neg\varphi \in \mathcal{L}_{\mathcal{Z}}$													•

Table 1. Van Glabbeek's logical characterizations for the semantics in the ltbt-spectrum

Therefore, by floating away any $\vee$ in a formula in $\mathcal{L}_Z^{\vee}$, it becomes equivalent to a disjunction of formulas within the corresponding language $\mathcal{L}_Z$, and then the equivalence of both logics follows.

Note that $\wedge$ cannot be filtered by the prefix operator. By the way this fact generates the difference between linear semantics (whose logics do not allow an arbitrary use of conjunction) and branching semantics (where we can arbitrarily use conjunction).

2.2 Observational characterizations for process semantics

Since we will relate at the end of the paper our new logical characterizations with the unified observational characterizations of the semantics from [dFGP09b], we briefly present next the definitions needed to get these observational characterizations.

One important fact about these characterizations is its finite character. All the considered observations are finite, and this means that the characterizations work as long as we keep ourselves to the continuous side of the range of possible semantic domains. Therefore, we have to restrict ourselves to finite processes, or at least to image-finite processes. It is for this class of processes that Th. 1 works.

Definition 4 ([dFGP09b]). *The sets L_N of local observations corresponding to each of the N-constrained simulations in the spectrum, and $L_N(p)$ of observations associated to a process p, are defined as follows:*

- *S: $L_U = \{\cdot\}$, $L_U(p) = \cdot$.*
- *CS: $L_C = Bool$, $L_C(p)$ is true if $p \equiv \mathbf{0}$ and false otherwise.*
- *RS: $L_I = \mathcal{P}(Act)$, $L_I(p) = I(p) = \{a \mid a \in Act \text{ and } p \xrightarrow{a}\}$.*
- *TS: $L_T = \mathcal{P}(Act^*)$, $L_T(p)$ is T(p), the set of traces of p.*
- *2S: $L_S = \{\|p\|_S\}$, $L_S(p) = \|p\|_S$.*
- *NS: $L_S = \{\|p\|_{(n-1)S}\}$, $L_S(p) = \|p\|_{(n-1)S}$, where $\|p\|_{kS}$ denotes the k-nested simulation equivalence class of p.*

Definition 5 ([dFGP09b]).

1. *A branching general observation (bgo for short) of a process is a finite, non-empty tree whose arcs are labeled with actions in Act and whose nodes are labeled with local observations from L_N, for N a constraint; the corresponding set BGO_N is recursively defined as: $\langle l, \emptyset\rangle \in BGO_N$ for $l \in L_N$; $\langle l, \{(a_i, bgo_i) \mid i \in 1..n\}\rangle \in BGO_N$ for every $n \in \mathbb{N}, a_i \in Act$ and $bgo_i \in BGO_N$.*
2. *The set $BGO_N(p)$ of branching general observations of p corresponding to the constraint N is $BGO_N(p) = \{\langle L_N(p), S\rangle \mid S \subseteq \{(a, bgo) \mid bgo \in BGO_N(p'), p \xrightarrow{a} p'\}\}$.*
3. *We write $p \leq_N^b q$ if $BGO_N(p) \subseteq BGO_N(q)$.*

Theorem 1 ([dFGP09b]). *For all $N \in \{U, C, I, T, S\}$ and any two processes p and q, $p \sqsubseteq_{NS} q$ iff $p \leq_N^b q$.*

Definition 6 ([dFGP09b]).

1. *The set LGO_N of linear general observations (lgo for short) for a local observer L_N is the subset of BGO_N defined as: $\langle l, \emptyset\rangle \in LGO_N$ for each $l \in L_N$; $\langle l, \{(a, lgo)\}\rangle$ whenever $a \in Act$ and $lgo \in LGO_N$.*
2. *The set $LGO_N(p)$ of linear general observations of a process p with respect to the local observer L_N is $LGO_N(p) = BGO_N(p) \cap LGO_N$.*

Definition 7 ([dFGP09b]). *For $\zeta, \zeta' \subseteq LGO_N$, we define the orders $\leq_N^l$, $\leq_N^{l\supseteq}$, $\leq_N^{lf}$, and $\leq_N^{lf\supseteq}$ by:*

- $\zeta \leq_N^l \zeta' \Leftrightarrow \zeta \subseteq \zeta'$.
- $\zeta \leq_N^{l\supseteq} \zeta' \Leftrightarrow \forall\, X_0a_1X_1 \ldots X_n \in \zeta\ \exists\, Y_0a_1Y_1 \ldots Y_n \in \zeta'\ \forall i \in 0..n\ X_i \supseteq Y_i$.
- $\zeta \leq_N^{lf} \zeta' \Leftrightarrow \forall\, X_0a_1X_1 \ldots X_n \in \zeta\ \exists\, Y_0a_1Y_1 \ldots Y_n \in \zeta'\ X_n = Y_n$.
- $\zeta \leq_N^{lf\supseteq} \zeta' \Leftrightarrow \forall\, X_0a_1X_1 \ldots X_n \in \zeta\ \exists\, Y_0a_1Y_1 \ldots Y_n \in \zeta'\ X_n \supseteq Y_n$.

Definition 8 ([dFGP09b]). *Given two processes p and q we have $p \leq_N^Z q$ iff $LGO_N(p) \leq_N^Z LGO_N(q)$, where $Z \in \{l, l \subseteq, lf, lf \subseteq\}$. We will denote the corresponding equivalence by $=_N^Z$.*

Definition 9 ([dFGP09b]). *For all $\zeta \subseteq LGO_N$, we consider the following closures:*

- $\overline{\zeta}^{\supseteq} = \{X_0a_1X_1 \dots X_n \in \zeta \mid \exists\ Y_0a_1Y_1 \dots Y_n \in \zeta'\ \forall i \in 0..n\ X_i \supseteq Y_i\}$,
- $\overline{\zeta}^{f} = \{X_0a_1X_1 \dots X_n \in \zeta \mid \exists\ Y_0a_1Y_1 \dots Y_n \in \zeta'\ X_n = Y_n\}$,
- $\overline{\zeta}^{f\supseteq} = \{X_0a_1X_1 \dots X_n \in \zeta \mid \exists\ Y_0a_1Y_1 \dots Y_n \in \zeta'\ X_n \supseteq Y_n\}$.

Proposition 1 ([dFGP09b]). *For all $X \in \{\supseteq, f, f \supseteq\}$, $\zeta \leq_N^{lX} \zeta'$ if and only if $\overline{\zeta}^{X} \subseteq \overline{\zeta'}^{X}$.*

Recalling the classification presented in [dFGP09b], we can divide the new spectrum into four parts:

- Bisimulation semantics, which is characterized by HML as shown in [HM80] and [HM85]. Note that this logic is closed under negation ($\neg$), and therefore the preorder defined is an equivalence (the bisimulation). The remaining semantics are defined by non-trivial preorders, i.e., the preorders are not equivalences, and their logical characterizations are, of course, not closed under negation.
- Simulation semantics (S, CS, RS, ...), characterized by branching observations, which will be reflected by the non-restricted use of the operator $\bigwedge$ in the formulas.
- Linear semantics (T, F, R, ...), characterized by linear observations. We will get them by severely restricting the use of $\bigwedge$ and the use of the negation.
- Deterministic branching semantics, corresponding to an intermediate class between branching and linear semantics, where determinism appears restricting the use of the operator $\bigwedge$ in combination with the operator prefix. The only semantics in this class is PW.

3 A new logical characterization of the most popular semantics

Next we will present the new logics that characterize the different semantics in a uniform way. Following the same procedure that was used in [dFGP09a,dFGP09b], we will begin by studying the particular cases of the best known classical semantics, that is, those at the level of ready simulation in the extended spectrum. In Section 4, we will present the logics for the rest of the semantics in a unified way.

We will prove the equivalence between each of our logics and the corresponding logical characterization defined by van Glabbeek. In this way we will have proved that our new logical characterizations are indeed correct. But one of the intended goals of our unification of the semantics was the possibility of obtaining direct and natural proofs. This will be illustrated in Section 5 by showing the equivalence between each of our logical semantics and the corresponding observational semantics in [dFGP09b]. This provides a new proof of the correctness of these new characterizations without having to resort to the individual original proofs of correctness for each of the semantics.

In Figure 3 we recall the semantics that are more "popular" due to their "simplicity" and interest, then we start our unification work with them, as it was done in [dFGP09a]. We prefer to postpone the deterministic branched semantics in the figure, that is possible worlds, because its definition is a bit more complex.

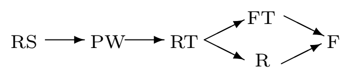

Fig. 3. More Popular Semantics

Definition 10 (The new logical characterizations). *We start with the characterization of ready simulation semantics, where, as already mentioned in the introduction, its branching character will be reflected by the unrestricted use of the $\bigwedge$ operator. Next, we consider the linear semantics at the level of ready simulation in the spectrum, that is, ready traces, failure traces, readiness and failures semantics.*

- ***Ready Simulation semantics(RS)****: we define the set of formulas $\mathcal{L}'_{RS}$ for ready simulation semantics by*
  - $\sigma \in \mathcal{L}_I \;\; \Rightarrow \sigma \in \mathcal{L}'_{RS}$;
  - $\sigma \in \mathcal{L}_I \;\; \Rightarrow \neg\sigma \in \mathcal{L}'_{RS}$;
  - $\varphi_i \in \mathcal{L}'_{RS} \; \forall i \in I \Rightarrow \bigwedge_{i \in I} \varphi_i \in \mathcal{L}'_{RS}$;
  - $\varphi \in \mathcal{L}'_{RS},\;\; a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_{RS}$.

  where $\mathcal{L}_I = \{a\top / a \in Act\}$.
- ***Ready traces semantics (RT)****: we define the set of formulas $\mathcal{L}'_{RT}$ for ready trace semantics by*
  - $\top \in \mathcal{L}'_{RT}$;
  - $\varphi \in \mathcal{L}'_{RT}, X_1, X_2 \subseteq \mathcal{L}'_I \Rightarrow (\bigwedge_{a \in X_1} a\top \wedge \bigwedge_{b \in X_2} \neg b\top) \wedge \varphi \in \mathcal{L}'_{RT}$;
  - $\varphi \in \mathcal{L}'_{RT},\;\; a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_{RT}$.
- ***Failure traces semantics (FT)****: we define the set of formulas $\mathcal{L}'_{FT}$ for failure traces semantics by*
  - $\top \in \mathcal{L}'_{FT}$;
  - $\varphi \in \mathcal{L}'_{FT}, X_1 \subseteq \mathcal{L}'_I \Rightarrow (\bigwedge_{a \in X_1} \neg a\top) \wedge \varphi \in \mathcal{L}'_{FT}$;
  - $\varphi \in \mathcal{L}'_{FT},\;\; a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_{FT}$.
- ***Readiness semantics (R)****: we define the set of formulas $\mathcal{L}'_R$ for readiness semantics by*
  - $\top \in \mathcal{L}'_R$;
  - $X_1 \subseteq \mathcal{L}'_I \;\; X_2 \subseteq \mathcal{L}'_I \Rightarrow (\bigwedge_{a \in X_1} a\top \wedge \bigwedge_{b \in X_2} \neg b\top) \in \mathcal{L}'_R$;
  - $\varphi \in \mathcal{L}'_R,\;\; a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_R$.
- ***Failures semantics (F)****: we define the set of formulas $\mathcal{L}'_F$ for failures semantics by*
  - $\top \in \mathcal{L}'_F$;
  - $X_1 \subseteq \mathcal{L}'_I \Rightarrow (\bigwedge_{a \in X_1} \neg a\top) \in \mathcal{L}'_F$;

- $\varphi \in \mathcal{L}'_F,\ \ a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}'_F$.

In the definition above, we avoid introducing new operators which are in fact just syntactic sugar, as done instead by van Glabbeek in [vG01]. As a consequence, it is not necessary to use new definitions for the notion of satisfaction, and we automatically obtain that all these logics define semantics coarser than bisimulation semantics.

Proposition 2 $\mathcal{L}'_{RS} \subseteq \mathcal{L}_B$.

Now we can prove that each of the above logics, $\mathcal{L}'_X$, are supersets of the corresponding logics, $\mathcal{L}_X$, defined by van Glabbeek in [vG01] for $X \in \{RS, RT, FT, R, F\}$. For the cases of failure traces and failures semantics we obtain in fact that the two logics are the same once the syntactic sugar used by van Glabbeek is removed.

Proposition 3 *1.* $\mathcal{L}'_{RS} \supseteq \mathcal{L}_{RS}$. *We also have* $\mathcal{L}_{RS} \subsetneq \mathcal{L}'_{RS}$.
2. $\mathcal{L}'_{RT} \supseteq \mathcal{L}_{RT}$. *We also have* $\mathcal{L}_{RT} \subsetneq \mathcal{L}'_{RT}$.
3. $\mathcal{L}'_{FT}$*=desugared(*$\mathcal{L}_{FT}$*), where the desugaring function removes the syntactic sugar used in* $\mathcal{L}_{FT}$.
4. $\mathcal{L}'_R \supseteq \mathcal{L}_R$. *We also have* $\mathcal{L}_R \subsetneq \mathcal{L}'_R$.
5. $\mathcal{L}'_F$*=desugared(*$\mathcal{L}_F$*), where the desugared function removes the syntactic sugar used in* $\mathcal{L}_F$.

Proof.
- 1| To prove that $\mathcal{L}'_{RS} \supseteq \mathcal{L}_{RS}$, it is sufficient to show that each formula $X = \bigwedge_{a\in X} a\top \wedge \bigwedge_{b\notin X} \neg b\top$ corresponding to $X \subseteq Act$ belongs to $\mathcal{L}'_{RS}$. Both $a\top$ and $\neg b\top$ are in $\mathcal{L}'_{RS}$, and the combination of these formulas with the operator $\wedge$ is also in the set $\mathcal{L}'_{RS}$. To prove that $\mathcal{L}_{RS} \subsetneq \mathcal{L}'_{RS}$, it is sufficient to see that the formula $\neg b\top$ belongs to $\mathcal{L}'_{RS}$. However, this formula does not belong to the set $\mathcal{L}_{RS}$.
- 2| Similarly as we did in the case of ready simulation semantics, to prove that $\mathcal{L}'_{RT} \supseteq \mathcal{L}_{RT}$ it is sufficient to show that for every $X \subseteq Act$ and any $\varphi \in \mathcal{L}_{RT}$, the formula $(\bigwedge_{af\in X} a\top \wedge \bigwedge_{b\notin X} \neg b\top)\varphi$ belongs to $\mathcal{L}'_{RT}$. Note that the condition $b \notin X$ is equivalent to $b \in \overline{X}$, so taking $X_1 = X$ and $X_2 = \overline{X}$ we have that the considered formula belongs to $\mathcal{L}'_{RT}$. To prove that $\mathcal{L}_{RT} \subset \mathcal{L}'_{RT}$, it is sufficient to note that the formula $(\neg b\top) \wedge \varphi$ belongs to $\mathcal{L}'_{RT}$ by simply taking $X_1 = \emptyset$ and $X_2 = \{b\}$, but it does not belong to $\mathcal{L}_{RS}$.
- 3| In this case the result is trivial, since the definitions of $\mathcal{L}_{FT}$ and $\mathcal{L}'_{FT}$ are indeed the same, once the syntactic sugar is removed.
- 4| To prove that $\mathcal{L}'_R \supseteq \mathcal{L}_R$, it is sufficient to show that for every $X \subseteq Act$ the formula $(\bigwedge_{a\in X} a\top \wedge \bigwedge_{b\notin X} \neg b\top)$ belongs to $\mathcal{L}'_R$. Note that the condition $b \notin X$ is equivalent to $b \in \overline{X}$, so taking $X_1 = X$ and $X_2 = \overline{X}$ we have that the considered formula belongs to $\mathcal{L}'_R$. To check that $\mathcal{L}_R \subsetneq \mathcal{L}'_R$, it is sufficient to note that the formula $(\neg b\top)$ belongs to $\mathcal{L}'_R$ by simply taking $X_1 = \emptyset$ and $X_2 = \{b\}$, while it does not belong to $\mathcal{L}_R$.
- 5| Trivial, the definitions of $\mathcal{L}_F$ and $\mathcal{L}'_F$ are indeed the same, except for the fact that van Glabbeek uses some syntactic sugar that we preferred to omit.

We have seen in Section 1 that our logics intend to be as large as possible, to obtain more natural characterizations. This is why, in most of the cases, we have obtained a logic larger than that proposed by van Glabbeek. In order to prove the equivalences between ours and van Glabbeek's logics, we have to show that the new formulas that we included in our logics are in fact redundant. Therefore, they could be removed without modifying the expressive power of the logics.

Proposition 4 *We have that (1)* $\mathcal{L}_{RS} \sim \mathcal{L}'_{RS}$*; (2)* $\mathcal{L}_{RT} \sim \mathcal{L}'_{RT}$*; (3)* $\mathcal{L}_{FT} \sim \mathcal{L}'_{FT}$*; (4)* $\mathcal{L}_R \sim \mathcal{L}'_R$ *and (5)* $\mathcal{L}_F \sim \mathcal{L}'_F$*.*

Proof.

- 1| Just observe that any conjunction of formulas in $\mathcal{L}_I$ and negations of formulas in $\mathcal{L}_I$ can be obtained as the disjunction of the formulas X describing all the "compatible" offers. These are those including the positive and negative information in the corresponding conjunction, i.e., $a\top \sim \bigvee_{a \in X} X$; $\neg a\top \sim \bigvee_{a \notin X} X$. Then by applying Prop. 1, we obtain $\mathcal{L}'_{RS} \sim \mathcal{L}_{RS}$.
- 2| We have seen that the formulas in $\mathcal{L}_{RT}$ are particular cases of the formulas in $\mathcal{L}'_{RT}$, those that define the offers at the states along a computation (when we apply the second clause in the definition of $\mathcal{L}'_{RT}$ taking $X_2 = \overline{X_1}$) and define these computations by means of the prefix operator (when we apply the third clause in the definition of $\mathcal{L}'_{RT}$). Instead, our more general formulas $(\bigwedge_{a\top \in X_1} a\top \wedge \bigwedge_{b\top \in X_2} \neg b\top) \wedge \varphi$, where $\varphi \in \mathcal{L}'_{RT}$, could give us some partial information, combining both positive information $a\top \in X_1$ and negative information $b\top \in X_2$, which tell us that we are in an arbitrary state X, satisfying $X_1 \subseteq X \subseteq \overline{X_2}$. But as we did for the ready simulation semantics, we can replace these formulas by the disjunction of all the formulas describing any of these possible offers X. By repeating this procedure at each level of the formula, we finally obtain a disjunction of formulas in $\mathcal{L}_{RT}$. To conclude, it is enough to apply Prop. 1.
- 4| Note that van Glabbeek allowed in $\mathcal{L}_R$ only "normal form" formulas from $\mathcal{L}'_R$, which can give us information about the offers at the final state in a computation (when we apply the second clause in the definition of $\mathcal{L}'_R$) or simply define these computations by means of the prefix operator (when we apply the third clause in the definition of $\mathcal{L}'_R$). However, our more general formulas $(\bigwedge_{a\top \in X_1} a\top \wedge \bigwedge_{b\top \in X_2} \neg b\top)$, could also give us some partial information about the final state, which could be both positive $a\top \in X_1$ and negative $b\top \in X_2$. In the (allowed) case $X_1 \bigcap X_2 \neq \emptyset$ we have that the formula is unsatisfiable. Otherwise, we are offering the actions a corresponding to formulas $a\top$ in any $X \subseteq \mathcal{L}_I$, that satisfies $X_1 \subseteq X$ and $X \subseteq \overline{X_2}$, and we can replace again the corresponding formula by a disjunction of formulas in $\mathcal{L}_R$.

In the following, when we consider a logic $\mathcal{L}_Z$ and the index Z refers to some concrete semantics, as it is the case with RS, RT, FT, R, F above, by abuse of notation we will simply write $\sqsubseteq'_Z$ instead of $\sqsubseteq_{\mathcal{L}'_Z}$, when referring to the preorder induced by the logic $\mathcal{L}'_Z$.

Theorem 2. *1. The logical semantics defined by $\sqsubseteq'_{RS}$ is equivalent to the observational branching semantics defined by $\leq^b_I$, generated by the set of branching general observations BGO_I.*
2. The logical semantics $\sqsubseteq'_{RT}$ induced by the logic $\mathcal{L}'_{RT}$ is equivalent to the observational linear semantics defined by the domain of linear general observations LGO_I, ordered by the preorder $\leq^l_I$, defined at Def. 7.
3. The logical semantics $\sqsubseteq'_{FT}$ induced by the logic $\mathcal{L}'_{FT}$ is equivalent to the observational linear semantics defined by the domain of linear general observations LGO_I, ordered by $\leq^{l\supseteq}_I$.
4. The logical semantics $\sqsubseteq'_R$ induced by the logic $\mathcal{L}'_R$ is equivalent to the observational linear semantics defined by LGO_I, ordered by $\leq^{lf}_I$.
5. The logical semantics $\sqsubseteq'_F$ induced by the logic $\mathcal{L}'_F$ is equivalent to the observational linear semantics defined by LGO_I, ordered by $\leq^{lf\supseteq}_I$.

Proof. It is an immediate consequence of Th. 1, Prop. 4 and the results by van Glabbeek collected in Table 1.

- 1| We have seen that our formulas are equivalent to van Glabbeek's formulas, $\mathcal{L}'_{RS} \sim \mathcal{L}_{RS}$. It is easy to show that once we have eliminated the unsatisfiable formulas in $\mathcal{L}'_{RS}$ (those that simultaneously make two different offers, or perform an action that was not included in the corresponding offer) the rest of formulas in $\mathcal{L}'_{RS}$ admit a normal form in the language $\mathcal{N}(\mathcal{L_{RS}})$, which we define as follows:
  - $X \subseteq Act,\ \ \{a_i/i \in I\} \subseteq X,\ \ \varphi_i \in \mathcal{N}(\mathcal{L_{RS}})\ \ \Rightarrow\ \ (\bigwedge_{b\in X} b\top \wedge \bigwedge_{b\notin X} \neg b\top) \wedge \bigwedge_{i\in I} a_i\varphi_i \in \mathcal{N}(\mathcal{L_{RS}})$,
  - $\{a_i/i \in I\} \subseteq Act,\ \ \varphi_i \in \mathcal{N}(\mathcal{L_{RS}})\ \ \Rightarrow\ \ \bigwedge_{i\in I} a_i\varphi_i \in \mathcal{N}(\mathcal{L_{RS}})$.

  Within this set, consider the subset of formulas $\mathcal{CN}(\mathcal{L_{RS}})$, which can be generated using the first clause in the above definition. Therefore we can establish a isomorphism between this set of formulas $\mathcal{CN}(\mathcal{L_{RS}})$ and the set of possible branching general observations BGO_I. Moreover, it is easy to prove that if for every formula $\varphi \in \mathcal{CN}(\mathcal{L_{RS}})$, we define bgo_φ as the corresponding observation, we have $\varphi \models p \Leftrightarrow bgo_\varphi \in BGO_I(p)$, from which it immediately follows that $\mathcal{CN}(\mathcal{L_{RS}})$ characterizes the ready simulation semantics defined via BGO_I.
  Now, to conclude the proof is sufficient to show that $\mathcal{N}(\mathcal{L_{RS}})$ and $\mathcal{CN}(\mathcal{L_{RS}})$ are equivalent. Note that whenever we use the second clause in the definition of $\mathcal{N}(\mathcal{L_{RS}})$, we are ignoring the possibility of specifying the offer X at the state we are. As a consequence, the offer could be any satisfying $\{a_i/i \in I\} \subseteq X$, for the corresponding set $\{a_i/i \in I\}$. Then we can complete the associated formula $\bigwedge_{i\in I} a_i\varphi_i$ adding the disjunction $\bigvee_{\{a_i/i\in I\}\subseteq X}(\bigwedge_{b\in X} b\top \wedge \bigwedge_{b\notin X} \neg b\top)$. Floating all the disjunctions we obtain a disjunction of formulas in $\mathcal{N}(\mathcal{L_{RS}})$, which ends the proof.

- 2| We know that $\mathcal{L}'_{RT} \sim \mathcal{L}_{RT}$. It is easy to show that eliminating all the unsatisfiable formulas (those that simultaneously offer two different sets of actions, or perform an action a that is not included in the corresponding offer X) the rest of formulas in $\mathcal{L}'_{RT}$ admit a normal form in the language $\mathcal{N}(\mathcal{L}_{\mathcal{RT}})$, which we define as follows:

- $X \subseteq Act \;\; \Rightarrow \;\; (\bigwedge_{b\in X} b\top \wedge \bigwedge_{b\notin X} \neg b\top) \in \mathcal{N}(\mathcal{L}_{\mathcal{RT}})$,
- $X \subseteq Act, \;\; a \in X, \;\; \varphi \in \mathcal{N}(\mathcal{L}_{\mathcal{RT}}) \;\; \Rightarrow \;\; (\bigwedge_{b\in X} b\top \wedge \bigwedge_{b\notin X} \neg b\top) \wedge a\varphi \in \mathcal{N}(\mathcal{L}_{\mathcal{RT}})$,
- $\top \in \mathcal{N}(\mathcal{L}_{\mathcal{RT}})$,
- $a \in Act, \;\; \varphi \in \mathcal{N}(\mathcal{L}_{\mathcal{RT}}) \;\; \Rightarrow \;\; a\varphi \in \mathcal{N}(\mathcal{L}_{\mathcal{RT}})$.

As we did for the case of ready simulation, we could define the corresponding language of complete formulas $\mathcal{CN}(\mathcal{L}_{\mathcal{RT}})$. The formulas in $\mathcal{L}'_{RT}$ that we obtained in the proof of Prop. 4, for the case of RT, are indeed in $\mathcal{CN}(\mathcal{L}_{\mathcal{RT}})$, because any sub-formula give us some partial information about the offers at the corresponding state, which in the worst case could be empty. Therefore, when we translate this information to the language $\mathcal{L}'_{RT}$ we obtain a disjunction between complete formulas in $\mathcal{CN}(\mathcal{L}_{\mathcal{RT}})$. Easily we can establish the isomorphism between this set of formulas $\mathcal{CN}(\mathcal{L}_{\mathcal{RT}})$ and the domain LGO_I, and it is easy to prove that for every formula $\varphi \in \mathcal{CN}(\mathcal{L}_{\mathcal{RT}})$, if we define lgo_φ as the corresponding observation, we have $\varphi \models p \Leftrightarrow lgo_\varphi \in LGO_I(p)$, from which it follows immediately that $\mathcal{CN}(\mathcal{L}_{\mathcal{RT}})$ characterizes the ready simulation semantics defined via LGO_I. To conclude the proof we need to show that $\mathcal{N}(\mathcal{L}_{\mathcal{RT}}) \equiv \mathcal{CN}(\mathcal{L}_{\mathcal{RT}})$, but this is proved in an analogous way as it was done for $\mathcal{N}(\mathcal{L}_{\mathcal{RS}})$ and $\mathcal{CN}(\mathcal{L}_{\mathcal{RS}})$ above.

- 3| $\Rightarrow$| Let p and q such that $p \sqsubseteq'_{FT} q$. We will show that $p \leq_I^{l\supseteq} q$.
  Given an observation $X_0a_1X_1 \ldots a_nX_n \in LGO_I(p)$, we have a failure trace for the process $p \in \mathbb{P}$, $\overline{X_0}a_1\overline{X_1} \ldots a_n\overline{X_n}$. Now, we consider the formulas $\varphi_n = \bigwedge_{a\in \overline{X}} \neg a\top$; $\varphi_i = \bigwedge_{a\in \overline{X_i}} \neg a\top \wedge a_{i+1}\varphi_{i+1}$ with $i \in 0 \ldots n-1$, and we have that $p \models \varphi_0$. Therefore $q \models \varphi_0$, which means that $\overline{X_0}a_1\overline{X_1} \ldots a_n\overline{X_n}$ is a failure trace of q. Then, there is some $Y_0a_1Y_2 \ldots a_nY_n \in LGO_I(p)$ with $Y_i \bigcap \overline{X_i} = \emptyset \;\forall i = 0 \ldots n$, or equivalently $X_i \supseteq Y_i \;\forall i = 0 \ldots n$. As a result, $LGO_I(p) \leq_I^{l\supseteq} LGO_I(q)$, which means $p \leq_I^{l\supseteq} q$.
  $\Leftarrow$| Let us suppose that for all $X_0a_1X_1 \ldots a_nX_n \in LGO_I(p)$ there exists $Y_0a_1Y_1 \ldots\; a_nY_n \in LGO_I(q)$ such that $X_i \supseteq Y_i \;\forall i = 0 \ldots n$. We want to show that if $p \models \varphi$ then $q \models \varphi$, for all $\varphi \in \mathcal{L}'_{FT}$.
  Let $p \models \varphi$, we can decompose φ by means of a sequence of formulas, taking $\varphi = \varphi_n$, $\varphi_i = \bigwedge_{a\in X_2^i} \neg a\top \wedge a_i\varphi_{i-1}$ for $i \in 1 \ldots n$ and $\varphi_0 = \bigwedge_{a\in X_2^0} \neg a\top$. Therefore, $X_na_nX_{n-1} \ldots a_1X_0$ is a failure trace for the process p, so there exists $Z_na_nZ_{n-1} \ldots a_1Z_0 \in LGO_I(p)$ with $Z_i \bigcap X_i = \emptyset$, and applying that $p \leq_I^{l\supseteq} q$, there exists some $Y_na_nY_{n-1} \ldots a_1Y_0 \in LGO_I(q)$ with $Y_i \subseteq Z_i$, so that $Y_i \bigcap X_i = \emptyset$ and then we get $q \models \varphi_n$.

- 4| Using the result in the proof of Prop. 4, for the case of R, it is enough to show the result for the set of "normal form" formulas $\mathcal{N}(\mathcal{L}_{\mathcal{R}})$ defined by
  - $X \subseteq Act, \;\; \Rightarrow \;\; (\bigwedge_{b\in X} b\top \wedge \bigwedge_{b\notin X} \neg b\top) \in \mathcal{N}(\mathcal{L}_{\mathcal{R}})$,
  - $\top \in \mathcal{N}(\mathcal{L}_{\mathcal{R}})$,
  - $a \in Act, \;\; \varphi \in \mathcal{N}(\mathcal{L}_{\mathcal{R}}) \;\; \Rightarrow \;\; a\varphi \in \mathcal{N}(\mathcal{L}_{\mathcal{R}})$.

$\Rightarrow|$ Let p and q such that $p \sqsubseteq'_R q$. We will show $p \leq_I^{lf} q$.
Given an observation $X_0 a_1 X_1 \ldots a_n X_n \in LGO_I(p)$, it corresponds to the readiness information $a_1 \ldots a_n X_n$ of p. Now, we consider the formulas $\varphi_n = \bigwedge_{a \in X} a\top \wedge \bigwedge_{a \notin X} \neg a\top$; $\varphi_{i-1} = a_i \varphi_i$ with $i \in 1 \ldots n-1$, and we have that $p \models \varphi_0$. Therefore $q \models \varphi_0$, and $a_1 \ldots a_n X_n$ is a readiness information of q, as a consequence, there is an observation $Y_0 a_1 Y_2 \ldots a_n Y_n \in LGO_I(q)$ with $Y_n = X_n$, proving $p \leq_I^{lf} q$.
$\Leftarrow|$ Let us suppose that for all $X_0 a_1 X_1 \ldots a_n X_n \in LGO_I(p)$ there exists some $Y_0 a_1 Y_1 \ldots a_n Y_n \in LGO_I(q)$ such that $X_n = Y_n$. We want to show that if $p \models \varphi$ then $q \models \varphi$ for all $\varphi \in \mathcal{CN}(\mathcal{L}_{\mathcal{R}})$.
Let $p \models \varphi$, we can decompose φ taking $\varphi = \varphi_n$, $\varphi_i = a_i \varphi_{i-1}$, for all $i \in 1 \ldots n$, and $\varphi_0 = \bigwedge_{a \in X_0} a\top \wedge \bigwedge_{a \notin X_0} \neg a\top$. Then we have that $a_n a_{n-1} \ldots a_1 X_0$ is a readiness information of p, so there exists some $Z_n a_n Z_{n-1} \ldots a_1 X_0 \in LGO_I(p)$, and then exists some $Y_n a_n Y_{n-1} \ldots a_1 Y_0 \in LGO_I(q)$ with $Y_0 = X_0$, from which we conclude that $q \models \varphi_n$.

- 5| $\Rightarrow|$ Let p and q such that $p \sqsubseteq'_F q$. We will show $p \leq_I^{lf\supseteq} q$.
Given an observation $X_0 a_1 X_1 \ldots a_n X_n \in LGO_I(p)$, it generates a (maximal) failure $a_1 \ldots a_n \overline{X_n}$ of the process p. Now, we consider the formulas $\varphi_n = \bigwedge_{a \in \overline{X}} \neg a\top$; $\varphi_{i+1} = a_{i+1} \varphi_i$ with $i \in 0 \ldots n-1$, and we have that $p \models \varphi_0$. Therefore, $q \models \varphi_0$, so $a_1 \ldots a_n \overline{X_n}$ is a failure information of q, and then there is some $Y_0 a_1 Y_2 \ldots a_n Y_n \in LGO_I(q)$ with $Y_n \bigcap \overline{X_n} = \emptyset$, or equivalently $X_n \supseteq Y_n$, proving that $p \leq_I^{lf\supseteq} q$.
$\Leftarrow|$ Let us suppose that for all $X_0 a_1 X_1 \ldots a_n X_n \in LGO_I(p)$ there exists some $Y_0 a_1 Y_1 \ldots a_n Y_n \in LGO_I(q)$ such that $X_n \supseteq Y_n$. We want to show that if $p \models \varphi$ then $q \models \varphi$ for all $\varphi \in \mathcal{L}'_F$.
Let $p \models \varphi$, we can decompose φ taking $\varphi = \varphi_n$, $\varphi_i = a_i \varphi_{i-1}$, with $i \in 1 \ldots n$, and $\varphi_0 = \bigwedge_{a \in X_0} \neg a\top$. From $p \models \varphi$ we infer that $a_n a_{n-1} \ldots a_1 X_0$ is a failure information of the process p, so there exists $Z_n a_n Z_{n-1} \ldots a_1 Z_0 \in LGO_I(p)$ with $Z_0 \bigcap X_0 = \emptyset$, and then there is some $Y_n a_n Y_{n-1} \ldots a_1 Y_0 \in LGO_I(q)$ with $Y_n \subseteq Z_n$, so that $Y_n \bigcap X_n = \emptyset$ obtaining $q \models \varphi_n$.

Example 1. Figure 2 shows the relationship between all the semantics in the "level" of RS. Figure 4 shows a collection of paradigmatic examples to illustrate differences between these semantics. These examples are "borderline", presenting pairs of processes that are equivalent w.r.t. any semantics in the spectrum that is coarser than the considered one. All these facts can be checked by taking any arbitrary formula from the logic characterizing each of the semantics. Moreover, in all the cases but the third and last ones, the two processes are ordered from left to right by $\sqsubseteq_{\mathcal{L}'_Z}$ for the coarsest preorder for which the equivalence fails. For readability, we omit the last $\top$ in all sub-formulas. Besides, $\sim_X$, (resp. $\not\sim_X$) , where X is a set of indexes, represents any $\sim_Z$ (resp. $\not\sim_Z$), with $Z \in X$.

- $P_1 \not\sim_F P_2$, and as a consequence $P_1 \not\sim_{\{R,\, FT,\, RT,\, RS\}} P_2$; we can check it using the fact that $P_1 \models ac(\neg d \wedge \neg e)$, but $P_2 \not\models ac(\neg d \wedge \neg e)$.

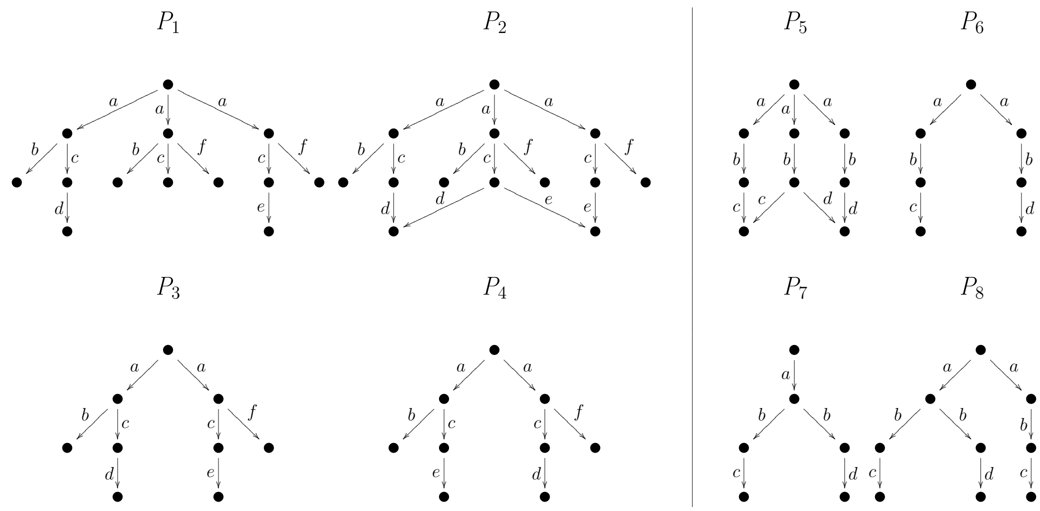

Fig. 4. Example to show the strength of the different logics

- $P_2 \sim_F P_3$; however, $P_2 \nsim_{\{R,\ RT,\ FT,\ RS\}} P_3$, using that $P_2 \models a(b \wedge c \wedge f)$, but $P_3 \not\models a(b \wedge c \wedge f)$.
- $P_3 \sim_{\{F,\ R\}} P_4$; however, $P_3 \nsim_{\{RT,\ FT,\ RS\}} P_4$, using that $P_3 \models a(\neg b)c(\neg d)$, but $P_4 \not\models a(\neg b)c(\neg d)$.
- $P_5 \sim_{\{F,\ FT\}} P_6$; however, $P_5 \nsim_{\{R,\ RT,\ RS\}} P_6$, using that $P_5 \models ab(c \wedge d)$, but $P_6 \not\models ab(c \wedge d)$.
- $P_6 \sim_{\{F,\ R,\ RT,\ FT\}} P_7$; however, $P_6 \nsim_{RS} P_7$, using that $P_7 \models a(bc \wedge bd)$, but $P_6 \not\models a(bc \wedge bd)$.
- $P_7 \sim_{\{F,\ R,\ RT,\ FT,\ RS\}} P_8$.

4 Our new unified logical characterizations of the semantics

Inspired by the concrete representative examples presented in the previous section, now we can define the general format for the logical characterization of each of the semantics in the *new spectrum*. We start by enlarging the spectrum a bit more to include all the elements needed to characterize the rest of the semantics in a very systematic way.

Definition 11. *1.* ***Universal semantics (U)****: We define the set of Universal formulas, $\mathcal{L}'_U$, that characterizes the trivial semantics that identifies all the processes, by $\top \in \mathcal{L}'_U$.*

2. ***Complete semantics (C)****: The Complete semantics is that defined by $\sqsubseteq_C$, taking $p \sqsubseteq_C q$ ::= ($p \xrightarrow{a} \Rightarrow \exists\ b \in Act\ q \xrightarrow{b}$). That is, it only distinguishes terminated processes (equivalent to* ***0****) from non-terminated ones. We define the set of Complete formulas $\mathcal{L}'_C$ characterizing it, by $\top,\ \neg 0 \in \mathcal{L}'_C$.*

3. ***Initial offer semantics (I)****: The Initial offer semantics is that defined by* $\sqsubseteq_I$*, taking* $p \sqsubseteq_I q ::= I(p) \subseteq I(q)$*. That is, it only observes the set of initial actions of each process,* $I(p) = \{a \mid a \in Act \wedge p \xrightarrow{a}\}$*. We define the set of Initial offer formulas* $\mathcal{L}'_I$ *characterizing it, , by* $\top, \neg 0 \in \mathcal{L}'_I$*;* $a\top \in \mathcal{L}'_I$ *for all* $a \in \mathrm{Act}$*.*

In the definition above the sub-formula $\neg 0$ is just syntactic sugar for the formula $\neg(\bigwedge_{a \in Act} \neg a\top)$, which can also be written as $\bigvee_{a \in Act} a\top$. Therefore, all these new logics are indeed sub-logics of $\mathcal{L}_B$, and so we do not need to define their semantics.

Note that when studying the particular case $N = I$ in Section 3, we already introduced, in Def. 10, a set of formulas $\mathcal{L}_I$, which is indeed equivalent to the (unified) logic $\mathcal{L}'_I$. This is so because the formula $\neg 0$ that we now include in $\mathcal{L}'_I$ is actually redundant. The main reason for which we prefer the larger logic $\mathcal{L}'_I$ is that we are trying to find in all the cases the largest possible logics. In fact, as a consequence of our choice, we immediately obtain that the Complete semantics is coarser than the Initial offer semantics, because $\mathcal{L}'_C \subseteq \mathcal{L}'_I$. Based on this result we will also easily obtain later, that the complete simulation is coarser than the ready simulation.

4.1 The simulation semantics

As discussed in [dFGP09b], the simulation semantics constitute the spine of the new spectrum. Moreover, all of them are defined in a homogeneous way using the notion of constrained simulation in [dFEGR08].

We can obtain the five simulation semantics in the new spectrum by considering the constraint in the set $\{U, C, I, T, S\}$. Therefore, we have that S_U is equivalent to the classical simulation (**S**), S_C is equivalent to the complete simulation (**CS**), S_I is equivalent to the ready simulation (**RS**), S_T is equivalent to the trace simulation (**TS**), and S_S is equivalent to the 2-nested simulation(**2S**) (see [dFGP09b] for the details of the used notation).

Definition 12. *We define the set of formulas* $\mathcal{L}'_{S_N}$ *associated to N-constrained simulation, where* $N \in \{U, C, I, T, S\}$*, by*

- $\sigma \in \mathcal{L}'_N \;\; \Rightarrow \sigma \in \mathcal{L}'_{S_N}$;
- $\sigma \in \mathcal{L}'_N \;\; \Rightarrow \neg\sigma \in \mathcal{L}'_{S_N}$;
- $\varphi_i \in \mathcal{L}'_{S_N} \forall i \in I \Rightarrow \bigwedge_{i \in I} \varphi_i \in \mathcal{L}'_{S_N}$;
- $\varphi \in \mathcal{L}'_{S_N}, \;\; a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_{S_N}$.

Next we will check the equivalence between our logical characterizations and those presented by van Glabbeek in [vG01].

Proposition 5 *We have* $\mathcal{L}'_S = \mathcal{L}_S$*.*

Proof. We can see that the clauses defining $\mathcal{L}'_S$ and $\mathcal{L}_S$ produce the same set of formulas. Although in $\mathcal{L}'_S$ we have the clauses $\sigma \in \mathcal{L}'_U \Rightarrow \sigma \in \mathcal{L}'_S$; $\sigma \in \mathcal{L}'_U \Rightarrow \neg\sigma \in \mathcal{L}'_S$, they produce two trivial formulas $\top$ and $\neg\top$, because in $\mathcal{L}'_U$ we only have the formula $\top$.

Proposition 6 *We have (1)* $\mathcal{L}'_{CS} = \mathcal{L}_{CS}$ *and (2)* $\mathcal{L}'_{2S} = \mathcal{L}_{2S}$*.*

Proof.

- 1| Once again, the logical sets of formulas produced by $\mathcal{L}'_{CS}$ and $\mathcal{L}_{CS}$ are the same. Although in $\mathcal{L}'_{CS}$, we have the clauses $\sigma \in \mathcal{L}'_C \Rightarrow \sigma \in \mathcal{L}'_{CS}$, $\sigma \in \mathcal{L}'_C \Rightarrow \neg\sigma \in \mathcal{L}'_{CS}$, from $\mathcal{L}'_C = \{\top, \neg\top\}$ we can only generate $\top$, $\neg\top$, 0 and $\neg 0$. We need in fact 0 to reflect the second clause in the definition of $\mathcal{L}_{CS}$, while $\neg 0 \equiv \bigvee_{a \in Act} a\top$ so $\neg 0$ is a disjunction of formulas from $\mathcal{L}_{CS}$, therefore any formula including it can be rewritten into a disjunction of formulas in $\mathcal{L}_{CS}$.
- 2| Another time, the logical sets of formulas produced by $\mathcal{L}'_{2S}$ and $\mathcal{L}_{2S}$ are the same. Although in $\mathcal{L}'_{2S}$, we have the clause: $\sigma \in \mathcal{L}'_S \Rightarrow \sigma \in \mathcal{L}'_{2S}$ it does not generate new logical formulas, because we have $\mathcal{L}_S \subseteq \mathcal{L}_{2S}$, since the formulas in $\mathcal{L}_S$ are exactly those that we could produce using only the last two clauses in the definition of $\mathcal{L}_{2S}$.

Remark 1. Note that we can use both affirmative formulas in $\mathcal{L}'_C$ and their negations. It comes from the fact that C-constrained simulation can be defined using an equivalence relation as constraint. However, it is also true that we could use $\sqsubseteq_C$ as a constraint, then we could remove the clause $\sigma \in \mathcal{L}'_C \Rightarrow \sigma \in \mathcal{L}'_{S_C}$, which generates $\neg 0 \in \mathcal{L}_{S_C}$, getting the same complete simulation semantics. It is important to note instead, that the other clause, which generates $0 \in \mathcal{L}'_{S_C}$, is crucial and cannot be removed from the definition.
These two facts appear also at the other simulation semantics in the extended spectrum, for which we also present a logic characterization including the two clauses above, but another equivalent characterization by removing one of those two clauses (but not the other!).

4.2 Logical characterization of the linear semantics

We start by defining the closure operators, by means of which we will be able to make precise to which extent conjunction can be used at the logical characterizations of each of the linear semantics.

Definition 13. *Given a logical set* $\mathcal{L}'_N$ *with* $N \in \{U, C, I, T, S\}$*, we define:*

1. ***Its symmetric closure*** $\mathcal{L}^{\equiv}_N$ *by:* $\sigma \in \mathcal{L}'_N \Rightarrow \sigma \in \mathcal{L}^{\equiv}_N$*;* $\sigma \in \mathcal{L}'_N \Rightarrow \neg\sigma \in \mathcal{L}^{\equiv}_N$*;* $\sigma_i \in \mathcal{L}^{\equiv}_N\ \forall i \in I \Rightarrow \bigwedge_{i \in I} \sigma_i \in \mathcal{L}^{\equiv}_N$*.*
2. ***Its negative closure*** $\mathcal{L}^{\neg}_N$ *by:* $\sigma \in \mathcal{L}'_N \Rightarrow \neg\sigma \in \mathcal{L}^{\neg}_N$*;* $\sigma_i \in \mathcal{L}^{\neg}_N\ \forall i \in I \Rightarrow \bigwedge_{i \in I} \sigma_i \in \mathcal{L}^{\neg}_N$*.*
3. ***Its positive closure*** $\mathcal{L}^{\surd}_N$ *by:* $\sigma \in \mathcal{L}'_N \Rightarrow \sigma \in \mathcal{L}^{\surd}_N$*;* $\sigma_i \in \mathcal{L}^{\surd}_N\ \forall i \in I \Rightarrow \bigwedge_{i \in I} \sigma_i \in \mathcal{L}^{\surd}_N$*.*

Remark 2. Obviously these closures can be defined for any given logic $\mathcal{L}$, but we prefer to use the particular case of $\mathcal{L}'_N$ since it will be enough for our goals here and it allows to use a simpler notation.

Now we can describe in a generic way all the linear semantics in the *new spectrum.*

Definition 14. *1. Based on the order $\leq^{l}_{N}$ we define the set of formulas $\mathcal{L}'_{\leq^{l}_{N}}$, by:*

- $\top \in \mathcal{L}'_{\leq^{l}_{N}}$;
- $\varphi \in \mathcal{L}'_{\leq^{l}_{N}},\ \ \sigma \in \mathcal{L}^{\equiv}_{N} \ \ \Rightarrow \sigma \wedge \varphi \in \mathcal{L}'_{\leq^{l}_{N}}$;
- $\varphi \in \mathcal{L}'_{\leq^{l}_{N}},\ \ a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_{\leq^{l}_{N}}$.

2. Based on the order $\leq^{l\supseteq}_{N}$ we define the set of formulas $\mathcal{L}'_{\leq^{l\supseteq}_{N}}$, by:

- $\top \in \mathcal{L}'_{\leq^{l\supseteq}_{N}}$;
- $\varphi \in \mathcal{L}'_{\leq^{l\supseteq}_{N}},\ \ \sigma \in \mathcal{L}^{\neg}_{N} \ \ \Rightarrow \sigma \wedge \varphi \in \mathcal{L}'_{\leq^{l\supseteq}_{N}}$;
- $\varphi \in \mathcal{L}'_{\leq^{l\supseteq}_{N}},\ \ a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_{\leq^{l\supseteq}_{N}}$.

3. Based on the order $\leq^{lf}_{N}$ we define the set of formulas $\mathcal{L}'_{\leq^{lf}_{N}}$, by:

- $\top \in \mathcal{L}'_{\leq^{lf}_{N}}$;
- $\sigma \in \mathcal{L}^{\equiv}_{N} \ \ \Rightarrow \sigma \in \mathcal{L}'_{\leq^{lf}_{N}}$;
- $\varphi \in \mathcal{L}'_{\leq^{lf}_{N}},\ \ a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_{\leq^{lf}_{N}}$.

4. Based on the order $\leq^{lf\supseteq}_{N}$ we define the set of formulas $\mathcal{L}'_{\leq^{lf\supseteq}_{N}}$, by:

- $\top \in \mathcal{L}'_{\leq^{lf\supseteq}_{N}}$;
- $\sigma \in \mathcal{L}^{\neg}_{N} \ \ \Rightarrow \sigma \in \mathcal{L}'_{\leq^{lf\supseteq}_{N}}$;
- $\varphi \in \mathcal{L}'_{\leq^{lf\supseteq}_{N}},\ \ a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_{\leq^{lf\supseteq}_{N}}$.

Note that for the coarsest semantics (e.g. those corresponding to plain refusals and plain readiness when $N = I$) we only observe N at the end of the formula. This is because we have no conjunctions in the formulas of the corresponding languages $\mathcal{L}'_{\leq^{lf}_{N}}$ and $\mathcal{L}'_{\leq^{lf\supseteq}_{N}}$, out those coming from the corresponding closures $\mathcal{L}^{\equiv}_{N}$ and $\mathcal{L}^{\neg}_{N}$. Instead, the other two logics introduce new conjunctions that allow the observation of N all along the computations. Moreover, we have used the negative closure at the "failures based" semantics, and the symmetric closure at the "readies based" semantics.

Definition 15. *We can use the positive closure to define two new semantics that were not studied in [dFGP09a,dFGP09b] nor elsewhere, as far as we know.*

1. *The semantics of* ***minimal trace offers*** *is that defined by the logic $\mathcal{L}'_{mto}$ with*
   - $\top \in \mathcal{L}'_{mto}$;
   - $\varphi \in \mathcal{L}'_{mto},\ \ \sigma \in \mathcal{L}^{\surd}_{N} \ \ \Rightarrow \sigma \wedge \varphi \in \mathcal{L}'_{mto}$;
   - $\varphi \in \mathcal{L}'_{mto},\ \ a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_{mto}$.
2. *The semantics of* ***minimal offers*** *is that defined by the logic $\mathcal{L}'_{mo}$ with*
   - $\top \in \mathcal{L}'_{mo}$;
   - $\sigma \in \mathcal{L}^{\surd}_{N} \ \ \Rightarrow \sigma \in \mathcal{L}'_{mo}$;
   - $\varphi \in \mathcal{L}'_{mo},\ \ a \in \mathrm{Act} \Rightarrow a\varphi \in \mathcal{L}'_{mo}$.

Although for each constraint N we have originally presented four different linear semantics, all of them collapse whenever the corresponding constraint is too simple, thus providing several characterizations of the same semantics. This is the case for $N = \{U, C\}$ which produces the trace semantics and the complete traces semantics, respectively. In these two cases it is easy to prove that the four logics are indeed equivalent. Therefore, we can use any of them to describe the trace semantics, in particular we will use the simplest characterizations to prove the following results.

Proposition 7 *We have (1)* $\mathcal{L}'_{\leq_U^{lf}} = \mathcal{L}_T$ *and (2)* $\mathcal{L}'_{\leq_C^{lf\supseteq}} = \mathcal{L}_{CT}$.

Proof.
- $\underline{1|}$ Trivial, since the sets of clauses defining $\mathcal{L}'_{\leq_U^{lf}}$ and $\mathcal{L}_T$ are nearly the same. Although we have in our definition the clause: $\sigma \in \mathcal{L}_U^{\equiv} \Rightarrow \sigma \in \mathcal{L}'_{\leq_U^{lf}}$ it does not add new logical formulas, because $\mathcal{L}_U^{\equiv} = \{\top\}$.
- $\underline{2|}$ Trivial, since the sets of clauses defining $\mathcal{L}'_{\leq_C^{lf\supseteq}}$ and $\mathcal{L}_{CT}$ are nearly the same. The only difference is that we use the general clause: $\sigma \in \mathcal{L}_C^{\neg} \Rightarrow \sigma \in \mathcal{L}'_{\leq_C^{lf\supseteq}}$ instead of adding directly the formula 0, but since $\mathbf{0} \in \mathcal{L}_C^{\neg}$, $\neg\mathbf{0} \in \mathcal{L}_C^{\neg}$ we have $0 \in \mathcal{L}'_{\leq_C^{lf\supseteq}}$.

Corollary 1. $\mathcal{L}'_{\leq_U^{lf}} \sim \mathcal{L}_T$ *and* $\mathcal{L}'_{\leq_C^{lf\supseteq}} \sim \mathcal{L}_{CT}$.

In order to illustrate the genericity of our characterizations it is interesting to consider one of the finest semantics in the classic spectrum. We are talking about the *Possible Future* semantics (PF). We find PF in Figure 1 below 2S, probably because the more accurate simulation semantics TS, was not (still) included in the spectrum. This is corrected in the new spectrum in Figure 2. Considering $N = T$, we have indeed the following result.

Proposition 8 *We have* $\mathcal{L}'_{\leq_T^{lf}} = \mathcal{L}_{PF}$.

Proof. Trivial, since the sets of clauses defining $\mathcal{L}'_{\leq_T^{lf}}$ and $\mathcal{L}_{PF}$ are nearly the same. The only difference is that in our definition we have the clause $\top \in \mathcal{L}'_{\leq_T^{lf}}$, but in the definition of $\mathcal{L}_{PF}$ did not explicitly appear, although it corresponds to the conjunction of an empty set of formulas.

Corollary 2. $\mathcal{L}'_{\leq_T^{lf}} \sim \mathcal{L}_{PF}$.

4.3 Logical characterization of the deterministic branching semantics

Next we consider the case of the deterministic branching semantics. In the classic spectrum the only such semantics is possible worlds (PW), but there is one such semantics for each level of the *new spectrum*.

In order to capture the determinism, we need to consider formulas which include conjunction to express the desired branching, but only when it corresponds to a choice between different actions. This leads us to the following scheme:

$$X \subseteq Act, \;\; \varphi_a \in \mathcal{L}_{D_N} \;\; \forall a \in X \Rightarrow \bigwedge_{a \in X} a\varphi_a \in \mathcal{L}_{D_N}$$

Definition 16. *For each $N \in \{U, C, I, T, S\}$, we define the formulas of $\mathcal{L}'_{D_N}$, by:*

- $\top \in \mathcal{L}'_{D_N}$;
- $\varphi \in \mathcal{L}'_{D_N}$, $\sigma \in \mathcal{L}^{\equiv}_{N} \;\Rightarrow \sigma \wedge \varphi \in \mathcal{L}'_{D_N}$;
- $X \subseteq \text{Act}$, $\varphi_a \in \mathcal{L}'_{D_N}$ $\forall a \in X \Rightarrow \bigwedge_{a \in X} a\varphi_a \in \mathcal{L}'_{D_N}$.

For $N = I$ we obtain the unified logical characterization of the PW semantics.

Proposition 9 *We have $\mathcal{L}'_{D_I} \supseteq \mathcal{L}_{PW}$.*

Proof. Analogous to the case of ready simulation semantics.

By the way, $\mathcal{L}'_{D_I}$ and $\mathcal{L}_{PW}$ are not equivalent, but this is caused by the fact that the original logical characterization $\mathcal{L}_{PW}$ was wrong. It can be checked, for instance, that taking $p = abc + a(bc + d) + ab$ and $q = a(bc + d) + ab$ we have $p \not\sim_{PW} q$, but $p \sim_{\mathcal{L}_{PW}} q$, since $\mathcal{L}_{PW}$ cannot "observe" the intermediate offer that makes the possible world abc different from those of q. Instead, the formula $\varphi \equiv a(\neg d \wedge bc) \in \mathcal{L}_{D_I}$ is enough to distinguish p and q, since we have $p \models \varphi$ and $q \not\models \varphi$.

Corollary 3. $\mathcal{L}'_{D_I} \sim \mathcal{L}_{PW}$.

Formulas \ Constraints ($\mathcal{N}$)	U	C	I	T	S	B
$\top \in \mathcal{L}'_{\mathcal{N}}$	•	•	•	•	ν	ν
$\neg 0 \in \mathcal{L}'_{\mathcal{N}}$		•	•	ν	ν	ν
$a \in Act \Rightarrow \; a\top \in \mathcal{L}'_{\mathcal{N}}$			•	ν	ν	ν
$\varphi \in \mathcal{L}'_{\mathcal{N}}$, $a \in Act \Rightarrow$ $a\varphi \in \mathcal{L}'_{\mathcal{N}}$				•	•	•
$\varphi_i \in \mathcal{L}'_{\mathcal{N}}\ \forall i \in I \Rightarrow$ $\bigwedge_{i \in I} \varphi_i \in \mathcal{L}'_{\mathcal{N}}$					•	•
$\varphi \in \mathcal{L}'_{\mathcal{N}} \Rightarrow$ $\neg\varphi \in \mathcal{L}'_{\mathcal{N}}$						•

Table 2. Logical characterizations of the semantics used as constraints in the main N-constrained semantics

In Tables 2 and 3, we present schematically all our results in a three-dimensional way: Table 3, presents the rules defining the logics characterizing each of the semantics at some level of the new spectrum, while Table 2 contains the logics that characterize the constraint governing each of these "levels".

Formulas \ Semantics ($\mathcal{Y}_\mathcal{N}$)	$\leq_N^{\mathbf{lf}\supseteq}$	$\leq_N^{\mathbf{lf}}$	$\leq_N^{\mathbf{l}\supseteq}$	$\leq_N^{\mathbf{l}}$	D_N	S_N	$N \in \{U, C, I, T, S\}$
	F	R	FT	RT	PW	RS	$N = I$
$\top \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$	•	•	•	•	•	ν	
$\varphi \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$, $a \in Act \Rightarrow$ $a\varphi \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$	•	•	•	•	ν	•	
$\varphi \in \mathcal{L}_N^{\neg} \Rightarrow$ $\varphi \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$	•	ν	ν	ν	ν	ν	
$\varphi \in \mathcal{L}_N^{\equiv} \Rightarrow$ $\varphi \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$		•		ν	ν	ν	
$\varphi \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$, $\sigma \in \mathcal{L}_N^{\neg} \Rightarrow$ $\sigma \wedge \varphi \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$			•	ν	ν	ν	
$\varphi \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$, $\sigma \in \mathcal{L}_N^{\equiv} \Rightarrow$ $\sigma \wedge \varphi \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$				•	•	ν	
$X \subseteq Act$, $\varphi_a \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$ $\forall a \in X \Rightarrow$ $\bigwedge_{a \in X} a\varphi_a \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$					•	ν	
$\varphi_i \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$ $\forall i \in I \Rightarrow$ $\bigwedge_{i \in I} \varphi_i \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$						•	
$\varphi \in \mathcal{L}_N \Rightarrow$ $\varphi \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$						•	
$\varphi \in \mathcal{L}_N \Rightarrow$ $\neg\varphi \in \mathcal{L}'_{\mathcal{Y}_\mathcal{N}}$						•	

Table 3. Our new logical characterizations for the semantics at each level of the ltbt-spectrum

5 Relating the unified logical characterizations and the unified observational model

In this section we will prove directly the equivalence between our unified logical characterizations and the unified observational semantics developed in [dFGP09b]. As we indicated in Section 2 we have to restrict ourselves to finite image processes to obtain the result. As a byproduct, we obtain for this kind of processes that the finite parts of each of the corresponding languages, that are obtained by intersection with $\mathcal{L}^f_{HM}$, give us a pure finite logical characterization of the semantics.

We start by considering the following concept of normal formula.

Definition 17 (Normal formula $\mathcal{N}(\mathcal{L}^*)$).

1. *Given a set of formulas $\mathcal{L}^*$, whose outermost operator is not the conjunction, we define the set of induced normal formulas, $\mathcal{N}(\mathcal{L}^*)$, as those generated by the clause: If $\Gamma_1, \Gamma_2 \subseteq \mathcal{L}^*$, $\{a_i \mid i \in I\} \subseteq Act$, $\varphi_i \in \mathcal{N}(\mathcal{L}^*)$ then*

$$\Big(\bigwedge_{\sigma \in \Gamma_1} \sigma \wedge \bigwedge_{\sigma \in \Gamma_2} \neg\sigma\Big) \wedge \bigwedge_{i \in I} a_i\varphi_i \in \mathcal{N}(\mathcal{L}^*).$$

2. *Now, for each $N \in \{U, C, I, T, S\}$ and each $Y_N \in \{S_N, \leq_N^l, \leq_N^{l\supseteq}, \leq_N^{lf}, \leq_N^{lf\supseteq}, D_N\}$ in the spectrum, we define the set of normal formulas, $\mathcal{N}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N}) \subseteq \mathcal{L}'_{Y_N}$ simply as: $\mathcal{N}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N}) = \mathcal{N}(\mathcal{L}^*_\mathcal{N}) \bigcap \mathcal{L}'_{Y_N}$ where $\mathcal{L}^*_N$ is the set of formulas in $\mathcal{L}'_N$ whose outermost operator is not the conjunction.*

Remark 3. By abuse of notation when some of the elements that appear in our normal formulas do not appear in the corresponding set $\mathcal{L}'_{Y_N}$, we assume that these formulas have been extended by conjunction with $\top$, using the fact that $\bigwedge_{\sigma\in\varnothing}\sigma$ is another syntactic form to express $\top$.

Note that in these new sets of normal formulas, we admit the use of infinite conjunction. As a consequence, these formulas could also have infinite depth, if we consider the depths of the formulas in $\mathcal{L}'_N$. However, if we define the normal depth of formulas in $\mathcal{N}(\mathcal{L}^*_{\mathcal{N}})$ as that obtained by counting the recursive nesting in the application of Def. 17, then any normal formula has finite normal depth, and the set they form can be explored by structural induction.

Theorem 3. *The set of normal formulas $\mathcal{N}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}^*_{\mathcal{N}})$ associated to each of the semantics in the spectrum is equivalent to the full set of formulas $\mathcal{L}'_{Y_N}$.*

Proof. It is easy to see, by structural induction, that all the formulas in $\mathcal{L}'_{Y_N}$ admit a normal formula in the sense of Def. 17, that is obtained by regrouping the subformulas in the given formula and applying Prop. 1.

Definition 18. *We define the set of complete normal formulas $\mathcal{CN}(\mathcal{L}^*)$ (respectively, the set of complete normal formulas associated to each semantics in the spectrum $\mathcal{CN}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}^*_{\mathcal{N}})$) as the set of normal formulas (respectively the set of normal formulas associated to each semantics in the spectrum) that satisfy recursively the condition $\Gamma_2 = \overline{\Gamma_1}$.*

The next theorem proves that we can "approximate" any such infinite conjunction using finite conjunction and thus "real" formulas.

Theorem 4. *If we restrict ourselves to the class of finite image processes that are those that do not allow infinitely branching for any action $a \in Act$, any complete normal formula $\varphi \in \mathcal{CN}(\mathcal{L}^*)$ can be approximated by a set of finite normal formulas $\{\varphi^k \mid k \in \mathbb{N}\} \subseteq \mathcal{FN}(\mathcal{L}^*)$ that only use finite conjunction, that is, we have $p \models \varphi \Leftrightarrow p \models \varphi^k\ \forall k$.*

Proof. We define the sequence φ^n by structural induction on the normal depth of φ:

- $\underline{\varphi = (\bigwedge_{\sigma\in\Gamma_1}\sigma \wedge \bigwedge_{\sigma\in\overline{\Gamma_1}}\neg\sigma)}$ We consider a fixed enumeration of the set $\mathcal{L}^* = \{\sigma_k | k \in \mathbb{N}\}$, and we define $\mathcal{L}^{*\leqslant k} = \{\sigma_j \in \mathcal{L}^* | j \leqslant k\}$. Then, for each $n \in \mathbb{N}$, we define
$$\varphi^n = \bigwedge_{\sigma\in\Gamma_1\cap\mathcal{L}^{*\leqslant n}}\sigma \wedge \bigwedge_{\sigma\in\overline{\Gamma_1}\cap\mathcal{L}^{*\leqslant n}}\neg\sigma$$
It is clear that each φ^n informs us about $\sigma_n \in \mathcal{L}^*$ and then the result is immediate.
- $\underline{\varphi = (\bigwedge_{\sigma\in\Gamma_1}\sigma \wedge \bigwedge_{\sigma\in\overline{\Gamma_1}}\neg\sigma) \wedge \bigwedge_{i\in I} a_i\varphi_i}$ By structural induction we can assume that the result is true for any sub-formula φ_i. Then we define $\varphi^n = \bigwedge_{\sigma\in\Gamma_1\cap\mathcal{L}^{*\leqslant n}}\sigma \wedge \bigwedge_{\sigma\in\overline{\Gamma_1}\cap\mathcal{L}^{*\leqslant n}}\neg\sigma \wedge \bigwedge_{i\in I} a_i\varphi_i^n$. Now, if we decompose φ as

$\varphi_I \wedge \varphi_{II}$ (taking $\varphi_{II} = \bigwedge_{i\in I} a_i\varphi_i$, and analogously for the set of approximations) we have that $p \models \varphi^n \Leftrightarrow p \models \varphi_I^n \wedge p \models \varphi_{II}^n$. If $p \models \varphi^n$ then we have $p \models \varphi_I^n\ \forall n \in \mathbb{N}$, and arguing as in the base case above, we conclude $p \models \varphi_I$. Any finite image process p can be decomposed as: $p = \sum_{a_i\in Act} \sum_{j=1}^{m_i} a_i^j p_i^j$, and we have $p \models \varphi_{II}^n \Leftrightarrow \forall i\ \exists j\ a_i = a_i^j \wedge p_i^j \models \varphi_i^n$. Then if we have $p \models \varphi_{II}^n$ $\forall n \in \mathbb{N}$, for each i there exists some $j \in 1\ldots m_i$ such that $p_i^j \models \varphi_i^n$, for infinitely many n's, but this means that $p_i^j \models \varphi_i^n$, for all $n \in \mathbb{N}$, and then by applying the induction hypothesis we have $p_i^j \models \varphi_i$, thus getting $p \models \varphi$.

Theorem 5. *We can define a natural correspondence between the set of complete normal formulas associated to a semantics $\mathcal{CN}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}_\mathcal{N}^*)$ and the corresponding domain of observations that defines its observational semantics. That correspondence $\leftrightarrow$ satisfies that $\varphi \leftrightarrow \theta \Rightarrow (p \models \varphi \Leftrightarrow \theta \in Obs_s(p))$. Moreover, this correspondence produces the following results for each of the semantics in the spectrum:*

1. *The set of complete normal formulas $\mathcal{CN}_{\mathcal{S}_\mathcal{N}}(\mathcal{L}_\mathcal{N}^*)$ and the domain of branching general observations GBO_N are isomorphic.*
2. *The set of complete normal formulas $\mathcal{CN}_{\leq_N^l}(\mathcal{L}_\mathcal{N}^*)$ and the domain of linear general observations LGO_N are isomorphic.*
3. *The set of complete normal formulas $\mathcal{CN}_{\leq_N^{l\supseteq}}(\mathcal{L}_\mathcal{N}^*)$ and the domain of linear general observations LGO_N are isomorphic.*
4. *The set of complete normal formulas $\mathcal{CN}_{\leq_N^{lf}}(\mathcal{L}_\mathcal{N}^*)$ and the quotient domain $LGO_N/_{\simeq_N^{lf}}$ are isomorphic.*
5. *The set of complete normal formulas $\mathcal{CN}_{\leq_N^{lf\supseteq}}(\mathcal{L}_\mathcal{N}^*)$ and the quotient domain $LGO_N/_{\simeq_N^{lf\supseteq}}$ are isomorphic.*
6. *The set of complete normal formulas $\mathcal{CN}_{\mathcal{D}_\mathcal{N}}(\mathcal{L}_\mathcal{N}^*)$ and the domain of deterministic branching general observations $dBGO_N$ are isomorphic.*

Proof. • 1| As we can see in Figure 5, a branching observation is a bi-labelled tree, whose nodes are local observations and whose arcs are labelled by actions.

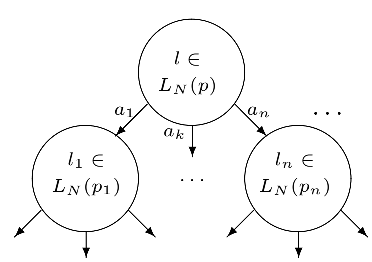

Fig. 5. General diagram of a *bgo*

The general form of any complete normal formula in $\mathcal{CN}_{\mathcal{S}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N})$ is $(\bigwedge_{\sigma\in\Gamma}\sigma\wedge\bigwedge_{\sigma\notin\Gamma}\neg\sigma)\wedge\bigwedge_{i\in I}a_if_i$, with $f_i\in\mathcal{CN}_{\mathcal{S}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N})$ $\forall i\in I$. Since the language $\mathcal{L}'_N$ characterizes the semantics used to get the local observations we can associate to each complete formula $(\bigwedge_{\sigma\in\Gamma}\sigma\wedge\bigwedge_{\sigma\notin\Gamma}\neg\sigma)$ the corresponding local observation $l\in L_N$ and then, by applying structural induction, we obtain the observation associated to each formula $f_i\in\mathcal{CN}_{\mathcal{S}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N})$, thus getting the branching general observation BGO_N associated to the given formula. It is easy to see that this correspondence is indeed a bijection.

- $\underline{2|}$ Analogous to Case 1, but in this case the obtained (degenerated) tree is just a single branch, thus corresponding to a lgo in LGO_N.
- $\underline{3|}$ In this Case, the general form of a complete normal formula in $\mathcal{CN}_{\leq^{l\supseteq}_N}(\mathcal{L}^*_\mathcal{N})$, is $f=(\top\wedge\bigwedge_{\sigma\notin\Gamma}\neg\sigma)\wedge af'$, with $f'\in\mathcal{CN}_{\leq^{l\supseteq}_N}(\mathcal{L}^*_\mathcal{N})$. If we close the set Γ by derivability obtaining Γ' and then consider its complement $\overline{\Gamma'}$, we can consider the local observation l that satisfies all the formulas in $\overline{\Gamma'}$ and none in Γ'. The linear general observation lgo corresponding to f, is then recursively defined as $\langle l,\{(a,lgo')\}\rangle$ where lgo' is the linear general observation corresponding to f'.
  To proceed in the opposite direction, we just need to take as Γ the complement of the set of formulas in $\mathcal{L}'_N$ satisfied by the local observation l at the root of the given LGO_N, and then we proceed in a recursive way.
- $\underline{4|}$ In this case, the general form of a complete normal formulas in $\mathcal{CN}_{\leq^{lf}_N}(\mathcal{L}^*_\mathcal{N})$ is $f=\top\wedge a_1(\ldots(\top\wedge a_{n-1}(\top\wedge a_n(\bigwedge_{\sigma\in\Gamma}\sigma\wedge\bigwedge_{\sigma\notin\Gamma}\neg\sigma)\ldots)$. Now we establish a correspondence between the set of local observations L_N and the sets $\Gamma\subseteq\mathcal{L}^*_N$ as done at Cases 1 and 2 above, and then we define the correspondence $\leftrightarrow$ by ignoring the values of all the intermediate local observations at the considered lgo, only considering the local observation at the end.
- $\underline{5|}$ We only need to apply the same procedure as in Case 4, using now the ideas in Case 3.
- $\underline{6|}$ Analogous to Case 1, but now it is not allowed to have repeated actions in the arcs leaving any node of an observation; this is obviously reflected at the form of the formulas in the corresponding language.

Remark 4. It is a bit surprising to find that the $lgo's$ in LGO_N are related in a bijective way both with the complete normal formulas in $\mathcal{N}_{\leq^l_N}(\mathcal{L}^*_\mathcal{N})$ and those in $\mathcal{N}_{\leq^{l\supseteq}_N}(\mathcal{L}^*_\mathcal{N})$. Let us consider the case $N=I$ to explain this fact. Then a cnf in $\mathcal{N}_{\leq^l_I}(\mathcal{L}^*_\mathcal{I})$ specifies the set at the corresponding local observation $I(p)\subseteq\mathcal{P}(Act)$ by means of a formula $(\bigwedge_{\sigma\in\Gamma}\sigma\wedge\bigwedge_{\sigma\notin\Gamma}\neg\sigma)$. The formulas in Γ are just the elements of the corresponding set $I(p)$, while those in $\overline{\Gamma}$ correspond to its complement (that gives redundant information here).

When considering the failure traces semantics, the formulas in $\mathcal{N}_{\leq^{l\supseteq}_I}(\mathcal{L}^*_\mathcal{I})$ only contain the negative part $\bigwedge_{\sigma\notin\Gamma}\neg\sigma$ that defines the complement of the adequate set $I(p)$. Since when working with this semantics the considered sets of $lgo's$ could be assumed to be closed w.r.t. the order $N^{l\supseteq}$ define in Def. 7, then we will not lose soundness when "assuming" that any formula $\bigwedge_{\sigma\notin\Gamma}\neg\sigma$ "generates" the observation associated to Γ, although it could be the case that some of the formulas $\sigma \in \Gamma$ were also not saisfied since the corresponding state $I(p)$ is smaller. But under the failures and failure traces semantics we can proceed by closing the set of offers upwards w.r.t. $\subseteq$ and no new failure is introduced.

Theorem 6. *The logical semantics $\sqsubseteq'_{Y_N}$ induced by the logic $\mathcal{L}'_{Y_N}$, where $Y_N \in \{S_N, \leq^l_N, \leq^{l\supseteq}_N, \leq^{lf}_N, \leq^{lf\supseteq}_N, D_N\}$, is equivalent to the corresponding observational semantics, defined at Def. 5 and Def. 6. In order to unify notation we will note here by GO_N the corresponding semantic domain.*

Proof. Using Th. 3 we get $\mathcal{L}'_{Y_N} \sim \mathcal{N}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N})$. Applying Th. 5 we get the isomorphism between the set $\mathcal{CN}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N})$ and the corresponding set of general observations GO_N.

To conclude the proof, we just need to show that $\mathcal{N}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N})$ and $\mathcal{CN}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N})$ are equivalent. Any consistent formula in $\mathcal{N}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N})$ ($\Gamma_1 \bigcap \Gamma_2 = \emptyset$), provides only some partial information about the states in a computation, so that the concrete values of these states are any characterized by a set Γ with $\Gamma_1 \subseteq \Gamma \subseteq \overline{\Gamma_2}$. Therefore, we can replace Γ_1 and Γ_2 by Γ and $\overline{\Gamma}$, respectively, adding the disjunction over all the possible values of Γ, to characterize the set of processes specified by the formula. Now it is enough to float the disjunction to obtain a disjunction of formulas in $\mathcal{CN}_{\mathcal{Y}_\mathcal{N}}(\mathcal{L}^*_\mathcal{N})$, and applying Prop. 1 and Th. 4, we get the equivalence between the two sets of formulas.

Corollary 4.
1. *The unified logical semantic defined at Def. 12 is equivalent to the N-simulation semantics.*
2. *The unified logical semantic defined at Def. 14.1 is equivalent to the N-ready traces semantics.*
3. *The unified logical semantic defined at Def. 14.2 is equivalent to the N-failure traces semantics.*
4. *The unified logical semantic defined at Def. 14.3 is equivalent to the N-readiness semantics.*
5. *The unified logical semantic defined at Def. 14.4 is equivalent to the N-failure semantics.*
6. *The unified logical semantic defined at Def. 16 is equivalent to the N-deterministic branched semantics.*

Proof. Since it was proved in [dFGP09b] that any observational semantics characterize the corresponding (classical) semantics in the (extended) ltbt-spectrum, we obtain as an immediate corollary the equivalence between our (unified) logical characterizations and the those classical semantics.

6 The real diamond structure

Now we will explore in more detail the real structure of the extended spectrum, as it was already done at the end of [dFGP09a]. One could think that each diamond in that spectrum corresponds to a lattice structure. However, this is not the case: there is another semantics coarser than both N-readiness and N-failure

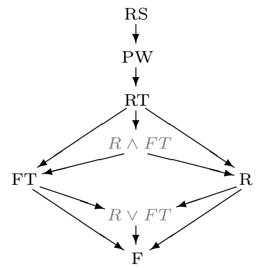

Fig. 6. The diamond below ready simulation

traces and finer than N-failures, and another finer than those two semantics and coarser than N-ready traces. Focusing on the case $N = I$ the obtained complete structure is that shown in Figure 6, in which we include the new join semantics $R \wedge FT$ and the meet one $R \vee FT$.

Since readiness semantics observes the ready set at the end of the trace, while failure traces observes failures during the computation, it is natural to expect that the join semantics $R \wedge FT$ will observe both failures during the computation and ready sets at the end. This is indeed the case. The corresponding observational characterization is obtained by means of a new order $\leq_N^{l\supseteq\wedge f}$ on the set LGO_I. Next we directly give the generic definition for any constraint N.

Definition 19. *Let* $\zeta, \zeta' \subseteq LGO_N$*, we define*

$$\zeta \leq_N^{l\supseteq\wedge f} \zeta' \Leftrightarrow \forall\ X_0 a_1 X_1 \dots X_n \in \zeta\ \exists\ Y_0 a_1 Y_1 \dots Y_n \in \zeta'\ (\forall i \in 0..n-1\ X_i \supseteq Y_i) \wedge\ X_n = Y_n$$

It is easy to see that we can also obtain $\leq_N^{l\supseteq\wedge f}$ as the conjunction of the orders $\leq_N^{l\supseteq}$ and $\leq_N^{lf}$, that is, $\zeta \leq_N^{l\supseteq\wedge f} \zeta' \Leftrightarrow \zeta \leq_N^{l\supseteq} \zeta'$ and $\zeta \leq_N^{lf} \zeta'$.

The observational characterization of the meet semantics $R \vee FT$ is a bit more complicated.

Definition 20. *Let* $\zeta, \zeta' \subseteq LGO_N$*, we define*

$$\zeta \leq_N^{l\supseteq\vee f} \zeta' \Leftrightarrow \forall\ X_0 a_1 X_1 \dots X_n \in \zeta\ \exists\ \{Y_0 a_1 Y_1 \dots Y_n^j | j \in J\} \subseteq \zeta' \text{ such that } X_n = \bigcup_{j\in J} Y_n^j$$

By means of some simple algebraic manipulations we can get the following equivalent expression:

$$\zeta \leq_N^{l\supseteq\vee f} \zeta' \Leftrightarrow \forall\ X_0 a_1 X_1 \dots X_n \in \zeta\ \forall a \in X_n\ \exists\ Y_0 a_1 Y_1 \dots Y_n \in \zeta' \text{ such that } (a \in Y_n \wedge Y_n \subseteq X_n)$$

Next we present the logical characterizations of these new semantics. Although these semantics were already included in the extended spectrum at [dFGP09b] to show the generality of our approach, and Roscoe has already studied the meet semantics $R \vee FT$ with the name of revivals semantics in [Ros09], we think that their logical characterizations show in a very clear way the features and properties of the new semantics. Since, obviously, the two new families of semantics are in the linear side of the spectrum, the corresponding families of formulas characterizing them will include both $\top$ and those formulas generated by applying the prefix operator: $\top \in \mathcal{L}_{\leq_N^Z}$; $\varphi \in \mathcal{L}_{\leq_N^Z}$, $a \in Act \Rightarrow a\varphi \in \mathcal{L}_{\leq_N^Z}$; where the super-index $Z \in \{\leq_N^{l\supseteq\wedge f}, \leq_N^{l\supseteq\vee f}\}$ will determine the two families of semantics.

Next we consider the case of the most popular semantics, i.e., those at the level of ready simulation (RS). Therefore, we have to characterize the semantics $R \wedge FT$ and $R \vee FT$. Obviously, the first corresponds to a semantics that is finer than both R and FT, and therefore the set of formulas defining its logical characterization will be bigger than those for these two semantics. In the second case we just need to connect the clauses that define those two logics in the adequate way.

Definition 21 (logical characterization of ($\mathbf{R} \wedge \mathbf{FT}$) and ($\mathbf{R} \vee \mathbf{FT}$)).

1. *We define the set of formulas $\mathcal{L}'_{\leq_I^{l\supseteq\wedge f}}$, as that generated by the clauses:*
   - $\top \in \mathcal{L}'_{\leq_I^{l\supseteq\wedge f}}$;
   - $\varphi \in \mathcal{L}'_{\leq_I^{l\supseteq\wedge f}}$, $\sigma \in \mathcal{L}_I^{\neg} \Rightarrow \sigma \wedge \varphi \in \mathcal{L}'_{\leq_I^{l\supseteq\wedge f}}$; $\sigma \in \mathcal{L}_I^{\equiv} \Rightarrow \sigma \in \mathcal{L}'_{\leq_I^{l\supseteq\wedge f}}$;
   - $\varphi \in \mathcal{L}'_{\leq_I^{l\supseteq\wedge f}}$, $a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}'_{\leq_I^{l\supseteq\wedge f}}$.
2. *We define the set of formulas $\mathcal{L}'_{\leq_I^{l\supseteq\vee f}}$ as that generated by the clauses:*
   - $\top \in \mathcal{L}'_{\leq_I^{l\supseteq\vee f}}$;
   - $\sigma, \sigma_j \in \mathcal{L}'_I \;\; \forall j \in J \Rightarrow (\sigma \wedge \bigwedge_{j\in J} \neg\sigma_j\top) \in \mathcal{L}_{\leq_I^{l\supseteq\vee f}}$;
   - $\varphi \in \mathcal{L}'_{\leq_I^{l\supseteq\vee f}}$, $a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}'_{\leq_I^{l\supseteq\vee f}}$.

Theorem 7. 1. *The logical semantics induced by the logic $\mathcal{L}'_{\leq_I^{l\supseteq\wedge f}}$ is equivalent to that defined by the order $\sqsubseteq_{\leq_I^{l\supseteq\wedge f}}$ induced by the observational semantics defined by LGO_I, with the order $\leq_I^{l\supseteq\wedge f}$.*
2. *The logical semantics induced by the logic $\mathcal{L}'_{\leq_I^{l\supseteq\vee f}}$ is equivalent to that defined by the order $\sqsubseteq'_{\leq_I^{l\supseteq\vee f}}$ induced by observational semantics defined by LGO_I, with the order $\leq_I^{l\supseteq\vee f}$.*

By the way, we just need to replace the constraint I by the generic constraint N to obtain the definitions and results for the general case.

7 Conclusions and future work

We have concluded in this paper the work on unification of all the strong process semantics by considering here the logic approach, while [dFGP09a,dFGP09b] considered the observational and the equational approaches. As in the previous cases, our main goal was to clarify the relationships between all the process semantics, that initially were classified in a slightly confused way in [vG01]. Our starting point has been the Henessy-Milner Logic [HM85]: we have looked for clearly structured parts of it, that characterize each of the semantics. Once more, the difference between branching-time semantics and linear-time semantics is the key point to isolate the components that generate that structure. Moreover, the formulas defining the constraint corresponding to each of the simulations semantics also appear in the definition of the languages characterizing each of the semantics at the same level in the spectrum.

We had expected that our unified logical semantics would be closer to the unified observational semantics than what we have finally found. This is because logical formulas do not have a natural structure, and then we have to impose it by introducing the adequate notions of normal forms. At the same time, this lack of structure allows to consider formulas defining approximation properties that, even if redundant, we have preferred to maintain in our logics in order to have simple syntactical ways to express those approximation properties.

Although we expected that our work would be mainly exploratory simply, classifying the semantics previously introduced in other papers, we have had at least a couple of surprises. First, we have "discovered" two new more linear semantics at each of the levels of the spectrum. They correspond in fact to the use of a part of the approximation properties discussed above. Additionally, we found out that the classic logical characterizations of the Possible Worlds (PW) semantics was wrong. We guess that this mistake was caused by a non-structured definition of the rules defining the logic; at least, it was when we were trying to unwrap the original characterization to look for the elements in our unified characterizations that we discovered the mistake.

Once that we have available all the unified characterizations of the semantics we will have a much clearer picture of the spectrum, and we can use the parameterized definitions to prove generic properties of all or a part of the semantics in a generic way, without having to repeat similar proofs for each of them.

There are several directions in which we plan to extend our work. Weak semantics are an obvious target: if there are indeed many strong process semantics, once we introduce internal actions an explosion occurs [vG93] and the unification work is even more necessary in order to clarify which are the most interesting semantics, and what the differences between them are. Another interesting direction comes from the combinations of logic and algebra as done by Luttgen and Vogler [LV10,LV09]. Again, we are interested in studying if their proposal is canonical or can be parameterized in some way in order to obtain other interesting combinations. Finally, a couple of papers [BC10,Gut09] have appeared recently, where the logical characterizations of the non-interleaving semantics are developed. Again, it would be interesting to look at these works in order to discover the key points that guide their characterizations and the possibility to combine them with the key points establishing the hierarchies discussed above.

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