On the unification of process semantics: observational semantics*

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Abstract. The complexity of parallel systems has produced a large collection of semantics for processes. Van Glabbeek's linear time-branching time spectrum provides a classification of most of these semantics; however, no suitable unified definitions were available. We have discovered how to unify them, both in an observational framework and in an equational framework. In this first part of our study we present the observational semantics, that stresses the differences between the simulation (branching) semantics and the extentional (linear) semantics. As a result we rediscover the classification in van Glabbeek's spectrum and shed light on it, obtaining a framework where we can consider all the semantics in the spectrum at the same time. Also, we have discovered some "lost links" that correspond to semantics, possibly not too interesting (at the moment), that provide a clearer picture of the spectrum.

1 Introduction

The complexity of parallel systems has given rise to a large collection of semantics for processes, whose diversity is mainly due to the way in which nondeterminism is treated. Most of these semantics have been compiled into van Glabbeek's linear-time branching-time (ltbt) spectrum [7], where they are first presented along the lines of their original definitions and are then characterized in three frameworks: observational/testing, logical, and by means of finite axiomatizations whenever possible. However, even when presented in a common framework these definitions are hard to compare with each other because the testing scenarios vary widely or the different sets of axioms appear completely unrelated.

After several years studying process semantics searching for homogeneous presentations we have discovered a way to unify them, both within an observational and an equational framework. Here we focus on the observational semantics, according to which we have classified the process semantics in four classes:

- bisimulation semantics, which is the only one that cannot be defined by means of a non-trivial preorder;

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– the simulation semantics (simulation, complete simulation, ready simulation, nested simulation, ...) characterized by means of branching observations, that is, labeled trees;
– the linear semantics (traces, failures, readiness, ...), characterized by linear observations, a degenerated case of branching observations;
– the deterministic branching semantics corresponding to an intermediate class between branching and linear, where observations are deterministic trees. Possible worlds semantics is the only semantics in the spectrum in this class.

Besides their linear or branching nature, semantics are characterized by a local observation function that generates the local observations of the states. For the linear case there is also the possibility of observing this local information in a partial way and this is how for each local observer, in principle, up to four different semantics can be obtained. In particular, this gives rise to the classic diamond below the ready simulation semantics formed by the failures, failure-traces, readiness, and ready-traces semantics.

Our uniform presentation of the process semantics will clarify the relations and hierarchies among them; moreover, it will make generic proofs of their properties possible. In particular, we have obtained a uniform presentation of their axiomatizations that we sketch in Appendix A and is studied in detail in a second part to this paper.

We are aware of the fact that there exists a very extensive literature on the field of semantics for concurrency and, in particular, on the ltbt spectrum but, due to lack of space we can only cite here some representative papers. Without any doubt [7], as commented above, is the key work on the subject, and in fact our guiding motivation is to complete it, providing a more uniform description of the ltbt spectrum; that work contains references to all the original presentations of the semantics in the spectrum. There have also been several efforts aimed at developing generic frameworks in which all those semantics could be uniformly presented. Since we are specially interested in the relation between the simulation (branching time) semantics and those based on decorated traces (linear time), we recall here the recent work by Jacobs [6] where he develops trace semantics in a coalgebraic framework, and that by Boreale and Gaducci [1], giving a coinductive presentation of failures semantics. We have also related the simulation semantics and the linear time semantics by introducing our (bi)simulations up-to [2], which have also shed light into the relation between the orders which are used to define the different semantics, and the equivalence relation induced by them [4]. We intend to include a section with historical notes and a comparison study in the joint extended version of our two papers, currently in preparation.

2 Preliminaries

Although the main results in this paper are valid for infinite processes, in order to simplify the presentation we will mainly consider finite processes generated by the basic process algebra BCCSP.

$$ap \xrightarrow{a} p \qquad \frac{p \xrightarrow{a} p'}{p+q \xrightarrow{a} p'} \qquad \frac{q \xrightarrow{a} q'}{p+q \xrightarrow{a} q'}$$

Fig. 1. Operational Semantics for BCCSP Terms

$$\begin{array}{llll} (B_1) & x + y \simeq y + x & (B_3) & x + x \simeq x \\ (B_2) & (x + y) + z \simeq x + (y + z) & (B_4) & x + \mathbf{0} \simeq x \end{array}$$

Fig. 2. Axiomatization for the (Strong) Bisimulation Equivalence

Definition 1. *Given a set of actions Act, the set BCCSP(Act) of processes is defined by the following BNF-grammar:*

$$p ::= \ \mathbf{0} \mid ap \mid p + q$$

where $a \in Act$; $\mathbf{0}$ represents the process that performs no action; for every action in Act, there is a prefix operator; and $+$ is a choice operator.

The operational semantics for BCCSP terms is defined in Figure 1. As usual, we write $p \xrightarrow{a}$ if there exists a process q such that $p \xrightarrow{a} q$.

Many different semantics for these non-deterministic processes have been defined in the literature. The most important and popular semantics appear in van Glabbeek's spectrum [7]. One indirect way to capture any semantics is by means of the equivalence relation induced by it: given a formal semantics $[\![\cdot]\!]_X$, we say that processes p and q are equivalent iff they have the same semantics, that is, $p \equiv_X q \Leftrightarrow [\![p]\!]_X = [\![q]\!]_X$. Also, these semantics can be defined by means of adequate observational scenarios, or by logical characterisations that introduce natural preorders $\sqsubseteq_X$ whose kernels are the semantic equivalences. For instance, we will write $\sqsubseteq_{RS}$ for ready simulation, $\sqsubseteq_F$ for failures, and so on. We refer to [7] for the original definition and usual notation for all the semantics in the ltbt spectrum that will be discussed throughout the paper.

Bisimilarity (denoted with $\equiv$), the strongest of the semantics in the spectrum, can be axiomatized by means of the four simple axioms in Figure 2. These axioms state that the choice operator is commutative, associative and idempotent, having the empty process as identity element. These axioms also justify the use of the notation $\sum_a \sum_i ap_a^i$ for processes, where the commutativity and associativity of the choice operator is used to group together the summands whose initial action is a.

The initial offer of a process is the set $I(p) = \{a \mid a \in Act \text{ and } p \xrightarrow{a}\}$. This is a simple, but quite important observation function that plays a central role in the definition of the most popular semantics in the linear time-branching time spectrum. We will also denote by I the relation expressing the fact that two processes have the same initial offer: $pIq \Leftrightarrow I(p) = I(q)$.

Some of the semantics in the spectrum are constrained simulation semantics that can be defined in a parameterized way.

Definition 2. *Given a relation N over BCCSP processes, a relation S_N is an* N-constrained simulation *if pS_Nq implies:*

- *For every a, if $p \xrightarrow{a} p'$ there exists q', $q \xrightarrow{a} q'$ and $p'S_Nq'$, and*
- *pNq.*

We say that process p is N-simulated by process q, or that q N-simulates p, written $p \sqsubseteq_{NS} q$, whenever there exists an N-constrained simulation S_N such that $p\ S_N\ q$.

We have already studied the constrained simulation semantics in detail in [3], stressing their general properties. In particular, the following constraints are considered: the universal relation U relating all processes, which gives rise to the simulation semantics; the relation C, which holds for processes p and q when both, or none, are isomorphic to $\mathbf{0}$, and that gives rise to the complete simulation semantics; I, which corresponds to ready simulation; T, that relates processes with the same traces and corresponds to trace simulation; S, the inverse of the simulation relation, whose associated constrained simulation is the 2-nested simulation.

Besides the semantics in the spectrum, we are interested in a general study covering any *reasonable* semantics coarser than bisimilarity. Since we will use preorders to characterise these semantics we introduce the following definitions that state the desired properties of those reasonable preorders.

Definition 3. *A preorder relation $\sqsubseteq$ over processes is a* behavior preorder *if*

- *it is weaker than bisimilarity, i.e. $p \equiv_B q \Rightarrow p \sqsubseteq q$, and*
- *it is a precongruence with respect to the prefix and choice operators, i.e. if $p \sqsubseteq q$ then $ap \sqsubseteq aq$ and $p + r \sqsubseteq q + r$.*

If $\sqsubseteq$ is actually an equivalence, it is called behavior equivalence*.*

3 Branching general observations

In order to characterize the simulation semantics in an extensional way we need local and branching general observations.

Definition 4. *The sets L_N of* local observations *corresponding to each of the N-constrained simulations in the spectrum, and $L_N(p)$ of observations associated to a process p, are defined as follows:*

- *Plain simulation: $L_U = \{\cdot\}$, $L_U(p) = \cdot$.*
- *Ready simulation: $L_I = \mathcal{P}(Act)$, $L_I(p) = I(p)$.*
- *Complete simulation: $L_C = Bool$, $L_C(p)$ is true if $p \equiv \mathbf{0}$ and false otherwise.*
- *Trace simulation: $L_T = \mathcal{P}(Act^*)$, $L_T(p)$ is $T(p)$, the set of traces of p.*

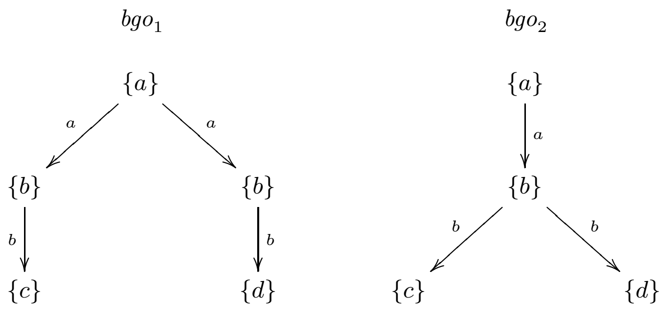

Fig. 3. Two branching observations

- *2-nested simulation:* $L_S = \{[\![p]\!]_S \mid p \in BCCSP\}$, $L_S(p) = [\![p]\!]_S$.
- *n-nested simulation:* $L_S = \{[\![p]\!]_{(n-1)S} \mid p \in BCCSP\}$, $L_S(p) = [\![p]\!]_{(n-1)S}$, *where* $[\![p]\!]_{kS}$ *denotes the k-nested simulation equivalence class of* p.

Definition 5. *1. A* branching general observation *(bgo for short) of a process is a finite, non-empty tree whose arcs are labeled with actions in Act and whose nodes are labeled with local observations from* L_N*, for* N *a constraint; the corresponding set* BGO_N *is recursively defined as:*
- $\langle l, \emptyset\rangle \in BGO_N$ *for* $l \in L_N$.
- $\langle l, \{(a_i, bgo_i) \mid i \in 1..n\}\rangle \in BGO_N$ *for every* $n \in \mathbb{N}$*,* $a_i \in Act$ *and* $bgo_i \in BGO_N$.

2. The set $BGO_N(p)$ *of branching general observations of* p *corresponding to the constraint* N *is*

$$BGO_N(p) = \{\langle L_N(p), S\rangle \mid S \subseteq \{(a, bgo) \mid bgo \in BGO_N(p'), p \xrightarrow{a} p'\}\} .$$

3. We write $p \leq_N^b q$ *if* $BGO_N(p) \subseteq BGO_N(q)$.

In Figure 3 some simple examples of bgo's for $N = I$ are shown. We represent bgo_1 as $\langle\{a\}, \{(a, \langle\{b\}, \{(b, \langle\{c\}, \emptyset\rangle)\}\rangle), (a, \langle\{b\}, \{(b, \langle\{d\}, \emptyset\rangle)\}\rangle)\}\rangle$ and bgo_2 as $\langle\{a\}, \{(a, \langle\{b\}, \{(b, \langle\{c\}, \emptyset\rangle), (b, \langle\{d\}, \emptyset\rangle)\}\rangle)\}\rangle$. We use braces for the set of children of a node, parentheses to represent a branch of the tree as a pair (initial arc, subtree below), and angle brackets to represent each tree as a pair $\langle$root, children$\rangle$.

Note that the bgo's of a process p described by its transition system can be generated by inductively applying the clauses defining the set $BGO_N(p)$, even when p is infinite. For instance, if $N = I$ and we consider the process $p ::= c.p$ defining a clock, since $\emptyset \subseteq \{(c, bgo) \mid bgo \in BGO_I(p), p \xrightarrow{c} p\}$, it follows that $\langle\{c\}, \emptyset\rangle \in BGO_I(p)$. But now $\{(c, \langle\{c\}, \emptyset\rangle)\} \subseteq \{(c, bgo) \mid bgo \in BGO_I(p), p \xrightarrow{c} p\}$ and therefore $\langle\{c\}, \{(c, \langle\{c\}, \emptyset\rangle)\}\rangle \in BGO_I(p)$, and so on.

It is clear that the bgo's of a process have an operational flavor. The nodes of the observations correspond to their states and the arcs to their transitions; this

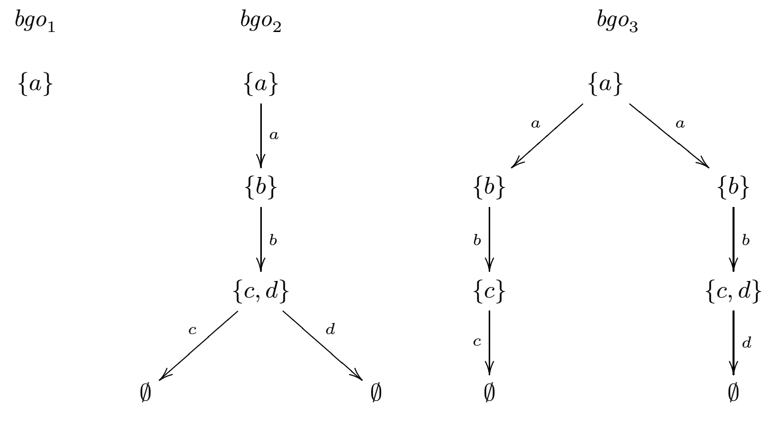

Fig. 4. Three branching observations

is why we will be able to define the orders associated to the different simulation semantics simply by set inclusion over the sets of bgo's.

Let us also comment on the fact that in all five cases that we have considered in Definition 4, which correspond to the five constrained simulations semantics in the spectrum, the local observation functions L_N define a representation of the equivalence relation N used to define the constrained simulation relations. This means that we have $L_N(p) = L_N(q) \Longleftrightarrow p\ N\ q$.

Example 1. For $N = I$, if $x = b(c+d)$ and $y = bc + bd$, then for $p = a(x+y)$ we have $bgo_k \in BGO_I(p)$ for $k \in \{1, 2, 3\}$, where the bgo's are depicted in Figure 4. It is easy to check that all of them are also branching observations of $q = a(x+y) + ax$. As a matter of fact, we have $BGO_I(p) = BGO_I(q)$. Note that in order to have $bgo_3 \in BGO_I(p)$ we need to consider two different observations of the process $x + y$, which is the only p' such that $a(x+y) \xrightarrow{a} p'$.

By contrast, for $p = a(bc + bd)$ and $q = abc + abd$, $BGO_I(q) \not\subseteq BGO_I(p)$, since for the branching observation bgo_1 in Figure 3 we have $bgo_1 \in BGO_I(q)$ and $bgo_1 \notin BGO_I(p)$. And also, we have $BGO_I(p) \not\subseteq BGO_I(q)$ since for bgo_2 as in Figure 3 we have $bgo_2 \in BGO_I(p)$ but $bgo_2 \notin BGO_I(q)$. The key idea is that, following our definition of bgo, we can include in a single bgo two separated computations but we cannot mix two different ones, even if the labels both in their initial transitions and in the local observations of the reached nodes were the same. This is why $bgo_2 \notin BGO_I(q)$.

The following simple properties will be immediate consequences of Theorem 2 below; we use them here to illustrate the expressive power of each kind of bgo.

Definition 6. *An axiom $p \preceq q$, respectively $p \simeq q$, is satisfied in a model BGO_N if $BGO_N(p') \subseteq BGO_N(q')$, respectively $BGO_N(p') = BGO_N(q')$, for every possible instantiation $p' \preceq q'$ or $p' \simeq q'$ of the axiom.*

Proposition 1. *1. The axiom (S) $x \preceq x + y$ is satisfied in the model BGO_U.*
2. The axiom ($S_\equiv$) $a(x + y) \simeq a(x + y) + ax$ is satisfied in the model BGO_U.

Proof. 1. It is an immediate consequence of the fact that if $p \xrightarrow{a} p'$ then $p + q \xrightarrow{a} p'$ and therefore $\{a \mid p \xrightarrow{a}\} \subseteq \{a \mid p + q \xrightarrow{a}\}$.
2. Again, it is a simple exercise to check that $BGO_U(p) \subseteq BGO_U(q)$ implies $BGO_U(ap) \subseteq BGO_U(aq)$, and that if $BGO_U(p), BGO_U(q) \subseteq BGO_U(r)$, then $BGO_U(p + q) \subseteq BGO_U(r)$; combining it with (1) produces the result. □

Proposition 2. *$BGO_I(p) \subseteq BGO_I(p + q)$ iff $I(q) \subseteq I(p)$.*

Proof. ($\Leftarrow$) Since $I(p + q) = I(p)$, the root of the bgo's is the same for both processes. ($\Rightarrow$) If $I(q) \not\subseteq I(p)$, then $I(p) \neq I(p + q)$ and the bgo's of p are not bgo's of $p + q$ because their roots contain fewer elements. □

The fact, that we now prove, that the observational semantics $BGO_N(p)$ can be defined in a compositional way is an important property that will simplify the proofs of many of their properties. Moreover, it also opens the door to the development of a presentation of the semantics in a purely denotational way, although we will not follow that road in this paper.

Theorem 1. *Let L be a function used as local observation function such that there exist semantical functions $+^L : L_N \times L_N \rightarrow L_N$ and $a^L : L_N \rightarrow L_N$ satisfying $L(ap) = a^L L(p)$ and $L(p + q) = L(p) +^L L(q)$. Then:*

- $BGO_N(ap) = \{\langle a^L L(p), \{(a, bgo) \mid bgo \in B\}\rangle \mid B \subseteq BGO_N(p)\}$.
- $BGO_N(p{+}q) = \{\langle L(p){+}^L L(q), S_1{\cup}S_2\rangle \mid \langle L(p), S_1\rangle \in BGO_N(p), \langle L(p), S_2\rangle \in BGO_N(q)\}$.

Proof. The first bullet is immediate by the definition of $BGO_N(ap)$. As for the second, we only need to realize that $p + q \xrightarrow{a} r$ iff $p \xrightarrow{a} r$ or $q \xrightarrow{a} r$: then, the children of $L_N(p + q)$ correspond to the union of the two sets associated to the transitions $p + q \xrightarrow{a} p_i$ and $p + q \xrightarrow{a} q_i$. Note that from the equalities above it follows that $BGO_N(p)$ can be computed compositionally. □

In particular, $BGO_N(p)$ is compositional for our fundamental constraints.

Proposition 3. *For all $N \in \{U, I, C, T, S\}$, L_N can be defined in a compositional way over the terms in BCCSP.*

Proof. Trivial, since all these observation functions correspond to semantics that can be defined denotationally. The result for U is obvious since it is a degenerate semantics that identifies all processes. Then, by Theorem 1 and Theorem 2 below we can conclude that the simulation semantics can indeed be denotationally defined. The result for traces is well-known, while I and C can be easily defined denotationally since $I(ap) = \{a\}$ and $I(p + q) = I(p) \cup I(q)$. □

Now we show that bgo's characterize N-simulation semantics in all cases.

Theorem 2. *For all* $N \in \{U, I, C, T, S\}$ *and any two processes* p *and* q, $p \sqsubseteq_{NS} q$ *iff* $p \leq_N^b q$.

Proof. ($\Rightarrow$) Let $p = \sum\sum ap_a^i$ and $q = \sum\sum aq_a^j$; if $p \sqsubseteq_{NS} q$, then $N(p,q)$ and therefore $L_N(p) = L_N(q)$. Now we proceed by induction on p. If $p \equiv \mathbf{0}$ the result is trivial. Otherwise, for every $a \in I(p)$ such that $p \xrightarrow{a} p_a$ there exists $q \xrightarrow{a} q_a$ such that $p_a \sqsubseteq_{NS} q_a$. By induction hypothesis $BGO_N(p_a) \subseteq BGO_N(q_a)$ from where, by the definition of $BGO_N(p)$, it follows that $BGO_N(p) \subseteq BGO_N(q)$.

($\Leftarrow$) Let us show that the relation $R = \{(p,q) \mid BGO_N(p) \subseteq BGO_N(q)\}$ is an N-simulation. If $(p,q) \in R$, then $L_N(p) = L_N(q)$ because $\langle L_N(p), \emptyset\rangle \in BGO_N(q)$ and thus $N(p,q)$. Now, for each $p \xrightarrow{a} p'$ we have $\{\langle L_N(p), \{(a, bgo)\}\rangle \mid bgo \in BGO_N(p')\} \subseteq BGO(q)$ and therefore there must exist some $q \xrightarrow{a} q'$ such that $BGO_N(p') \subseteq BGO_N(q')$, so that $(p', q') \in R$. ☐

Note that for this result to hold it is only required that the local observation function L_N satisfies $N(p,q)$ iff $L_N(p) = L_N(q)$. That is, L_N must compute a concrete representative of the equivalence class defined by N and this stresses again the interest of using behavior equivalences N as constraints for the definition of constrained simulations. Let us recall that, in principle, any behavior preorder could be used as such a constraint. For instance, the predicate $I_{\subseteq}$ defined by $I_{\subseteq}(p,q)$ iff $I(q) \subseteq I(p)$ could be used to define $I_{\subseteq}$-simulations (which coincide with I-simulations). But from $I(q) \subseteq I(p)$ we cannot conclude that $L_N(p) = L_N(q)$ and, hence, either a more complicated characterization of $\sqsubseteq_{NS}$ in terms of bgo's or an additional argument to show that $p \sqsubseteq_{I_{\subseteq}} q$ implies $I(p) \subseteq I(q)$ would be needed. And although this is obvious for a constraint as simple as I, or even T or S, it could be far from trivial for other, more complex constraints: therefore, it is always advisable to consider equivalence behaviors as constraints.

Corollary 1. *For any constraint* N *that is a behavior equivalence, whenever we have as local observation function* L_N *the quotient function* $L_N(p) = [\![p]\!]_N$ *or any concrete representation of it satisfying* $L_N(p) = L_N(q) \iff N(p,q)$*, then* $p \sqsubseteq_{NS} q$ *iff* $BGO_N(p) \subseteq BGO_N(q)$.

The results above bring forward the fact that despite the similarity between the bgo's of a process and its computation tree, the possibility of mixing several computations in a single branching observation makes it possible to identify non-bisimilar processes by their sets of branching observations.

4 Linear observations and linear time semantics

We introduce the linear observations of a process as a particular (degenerate) case of branching observations: those with a linear structure.

Definition 7. *1. The set* LGO_N *of* linear general observations *(lgo for short) for a local observer* L_N *is the subset of* BGO_N *defined as:*

– $\langle l, \emptyset\rangle \in LGO_N$ *for each* $l \in L_N$.
– $\langle l, \{(a, lgo)\}\rangle$, *whenever* $a \in A$ *and* $lgo \in LGO_N$.

2. *The set of* linear general observations *of a process* p *with respect to the local observer* L_N *is* $LGO_N(p) = BGO_N(p) \cap LGO_N$.

Since lgo's are linear they can be presented as traces, avoiding the sets of descendants in the general bgo's. Therefore, we will consider them as elements of the set $L_N \times (Act \times L_N)^*$.

It is also clear that the set of linear observations can be defined recursively without resorting to branching observations.

Proposition 4. *The set* $LGO_N(p)$ *of linear general observations of a process* p *is recursively defined by*

$$LGO_N(p) ::= \{\langle L_N(p)\rangle\} \cup \{\langle L_N(p), a\rangle \circ lgo \mid p \xrightarrow{a} p',\ lgo \in LGO_N(p')\}$$

We can also compute $LGO_N(p)$ in a compositional way.

Proposition 5. *Let* L *be a local observation function such that there exist semantical functions* $+^L : L_N \times L_N \rightarrow L_N$ *and* $a^L : L_N \rightarrow L_N$ *satisfying* $L(ap) = a^L L(p)$ *and* $L(p+q) = L(p) +^L L(q)$. *Then:*

– $LGO_N(ap) = \{\langle a^L L(p)\rangle\} \cup \{\langle a^L L(p), a\rangle \circ LGO_N(p)\}$.
– $LGO_N(p+q) = \{\langle L(p) +^L L(q)\rangle \circ t \mid \langle L(p)\rangle \circ t \in LGO_N(p)$ *or* $\langle L(p)\rangle \circ t \in LGO_N(q)\}$.

Proof. Just like Theorem 1. □

Obviously, for $N = U$ we have LGO_U isomorphic to Act^* and, thus, $LGO_U(p) = Traces(p)$. By contrast, for $N = I$, $LGO_I(p)$ is the set of ready traces of p, $ReadyTraces(p)$.

Set inclusion of the sets of linear observations with respect to a local observer L_N gives us the preorder defining the corresponding semantics.

Definition 8. *A process* p *is less than or equal to* q *with respect to the linear observations generated by* L_N, *denoted* $p \leq^l_N q$ *if* $LGO_N(p) \subseteq LGO_N(q)$. *We will denote the corresponding equivalence by* $=^l_N$.

Proposition 6. *(1)* $\leq^l_U = \sqsubseteq_T$; *(2)* $\leq^l_I = \sqsubseteq_{RT}$; *(3)* $\leq^l_C = \sqsubseteq_{CT}$.

Proof. It is trivial, since $LGO_U(p) = Traces(p)$, $LGO_I(p) = ReadyTraces(p)$, and $LGO_C(p) = \{(false, a_1) \circ \ldots \circ (false, a_n, true), (false, a_1) \circ \ldots \circ (false, a_i, false) \mid a_1, \ldots, a_n \in CompleteTraces(p), i < n\}$. □

Proposition 7. *For all* $N \in \{U, C, I, T, S\}$, *if* $p \sqsubseteq_{NS} q$ *then* $p \leq^l_N q$, *but the converse may not be true.*

Proof. The implication follows from Theorem 2 and the fact that lgo's are just a particular case of bgo's. For the converse, if we consider $N = U$ we have $\sqsubseteq_{US} = \sqsubseteq_S$ and $\leq^l_U = \sqsubseteq_T$, and it is well-known that $\sqsubseteq_S \not\subseteq \sqsubseteq_T$ since, for instance, $a(b+c) \not\sqsubseteq_S ab + ac$, but $a(b+c) =_T ab + ac$. □

Therefore, by means of linear observations and set inclusion, we can characterize the orders that define some of the semantics in the spectrum which are not simulation semantics. However, there are still some other semantics for which a different way of treating the linear observations is needed.

Definition 9. *For $\mathcal{T}, \mathcal{T}' \subseteq LGO_I$ we define the orders $\leq_I^{l\supseteq}$, $\leq_I^{lf}$, and $\leq_I^{lf\supseteq}$ by:*

- $\mathcal{T} \leq_I^{l\supseteq} \mathcal{T}' \iff \forall X_0 a_1 X_1 \ldots X_n \in \mathcal{T}\ \exists Y_0 a_1 Y_1 \ldots Y_n \in \mathcal{T}'\ \forall i \in 0..n\ X_i \supseteq Y_i$.
- $\mathcal{T} \leq_I^{lf} \mathcal{T}' \iff \forall X_0 a_1 X_1 \ldots X_n \in \mathcal{T}\ \exists Y_0 a_1 Y_1 \ldots Y_n \in \mathcal{T}'\ X_n = Y_n$.
- $\mathcal{T} \leq_I^{lf\supseteq} \mathcal{T}' \iff \forall X_0 a_1 X_1 \ldots X_n \in \mathcal{T}\ \exists Y_0 a_1 Y_1 \ldots Y_n \in \mathcal{T}'\ X_n \supseteq Y_n$.

Then, we write $p \leq_I^{lX} q$ if $LGO_I(p) \leq_I^{lX} LGO_I(q)$.

Since the definition of $\leq_I^{lf}$ ignores all the intermediate ready sets X_i with $i < n$ and requires the final ready sets to coincide, it defines the readiness preorder. Let us now prove that the two semantics based on failures are also characterized by our preorders $\leq_I^{lf\supseteq}$ and $\leq_I^{l\supseteq}$.

Proposition 8. *The preorder $\leq_I^{lf\supseteq}$ generates the failures preorder and $\leq_I^{l\supseteq}$ generates the failures trace preorder.*

Proof. The proof is based on the definition of initial failures of a process: p rejects X if and only if $X \cap I(p) = \emptyset$. Then, $\langle \alpha, X \rangle$ is a failure of p if and only if $p \stackrel{\alpha}{\Longrightarrow} p'$ and p' rejects X. Using lgo's, for $\alpha = a_1 \ldots a_n$, $\langle \alpha, X \rangle$ is a failure of p iff there exists $X_0 a_1 \ldots X_n \in \mathcal{T}$ such that $X_n \cap X = \emptyset$. Thus, $p \sqsubseteq_F p'$

iff $\mathit{Failures}(p) \subseteq \mathit{Failures}(p')$
iff $\langle \alpha, X \rangle \in \mathit{Failures}(p')$ for all $\langle \alpha, X \rangle \in \mathit{Failures}(p)$
iff $X_0 a_1 \ldots X_n \in LGO_I(p)$ with $X_n \cap X = \emptyset$ implies that there exists
$Y_0 a_1 \ldots Y_n \in LGO_I(p')$ with $Y_n \cap X = \emptyset$,

and then $p \leq_I^{lf\supseteq} p'$ implies $p \sqsubseteq_F p'$.

Conversely, assume that $p \sqsubseteq_F p'$ and recall that $p \leq_I^{lf\supseteq} p'$ iff for all $t = X_0 a_1 \ldots X_n \in LGO_I(p)$ there exists $Y_0 a_1 \ldots Y_n \in LGO_I(p')$ such that $X_n \supseteq Y_n$. If $t \in LGO_I(p)$ then $\langle \alpha, X_n^c \rangle \in \mathit{Failures}(p)$ and therefore $\langle \alpha, X_n^c \rangle \in \mathit{Failures}(p')$, which implies that there exists $p' \stackrel{\alpha}{\Longrightarrow} p''$ such that $I(p'') \cap X_n^c = \emptyset$, that is, there is $t' = Y_0 a_1 \ldots a_n I(p'') \in LGO_I(p')$ with $I(p'') \subseteq X_n$, and therefore we can conclude that $p \leq_I^{lf\supseteq} p'$.

The proof for failures traces is very similar and we omit it. □

As a matter of fact, the characterization of failures by means of the reverse inclusion of offerings is not a great discovery at all, and the same idea can be found in the definition of acceptance trees [5]. However, it is by means of our sets of linear observations that a quite nice characterization is obtained so that we can forget about the notion of failures and consider instead reverse inclusion of offerings. But the most important property of our characterizations in terms of different orders on the set LGO_I is that they can be generalized to other local observation functions.

Definition 10. *For* $\mathcal{T}, \mathcal{T}' \subseteq LGO_N$ *we define the orders* $\leq_N^{l\supseteq}$, $\leq_N^{lf}$, *and* $\leq_N^{lf\supseteq}$ *by:*

- $\mathcal{T} \leq_N^{l\supseteq} \mathcal{T}' \iff \forall X_0 a_1 X_1 \ldots X_n \in \mathcal{T}\ \exists Y_0 a_1 Y_1 \ldots Y_n \in \mathcal{T}'\ \forall i \in 0..n\ X_i \supseteq Y_i$.
- $\mathcal{T} \leq_N^{lf} \mathcal{T}' \iff \forall X_0 a_1 X_1 \ldots X_n \in \mathcal{T}\ \exists Y_0 a_1 Y_1 \ldots Y_n \in \mathcal{T}'\ X_n = Y_n$.
- $\mathcal{T} \leq_N^{lf\supseteq} \mathcal{T}' \iff \forall X_0 a_1 X_1 \ldots X_n \in \mathcal{T}\ \exists Y_0 a_1 Y_1 \ldots Y_n \in \mathcal{T}'\ X_n \supseteq Y_n$.

Then, we write $p \leq_N^{lX} q$ *if* $LGO_N(p) \leq_N^{lX} LGO_N(q)$.

By abuse of notation, we have used the superset inclusion symbol $\supseteq$ in the definitions above for all N. That is the right interpretation for the cases $N = I, T$; for $N = U, C$ the superset inclusions degenerate to equalities. For $N = S$, it should be interpreted as $[\![p]\!]_S \geq_S [\![q]\!]_S$. Then, it is easy to see that we could have used such an inequality $[\![p]\!]_N \geq_N [\![q]\!]_N$ in all cases.

For an observational semantics one expects that the order between processes is governed by set inclusion as is the case, for instance, for the classic definition of failures semantics. Fortunately, it is easy to obtain such a characterization for the three semantics considered above by means of suitable closure operators.

Definition 11. *For* $\mathcal{T} \subseteq LGO_N$, *the following three closures are defined:*

- $\overline{\mathcal{T}}^{\supseteq} = \{X_0 a_1 X_1 \ldots a_n X_n \mid \exists Y_0 a_1 Y_1 \ldots a_n Y_n \in \mathcal{T}\ \forall i \in 0..n\ X_i \supseteq Y_i\}$.
- $\overline{\mathcal{T}}^{f} = \{X_0 a_1 X_1 \ldots a_n X_n \mid \exists Y_0 a_1 Y_1 \ldots a_n X_n \in \mathcal{T}\}$.
- $\overline{\mathcal{T}}^{f\supseteq} = \{X_0 a_1 X_1 \ldots a_n X_n \mid \exists Y_0 a_1 Y_1 \ldots a_n Y_n \in \mathcal{T}\ X_n \supseteq Y_n\}$.

Then, if $X \in \{\supseteq, f, f\supseteq\}$, *for* $p \in BCCSP$ *and* N *a constraint, we define* $LGO_N^X(p) = \overline{LGO_N(p)}^X$.

Proposition 9. *All the operations in Definition 11 are indeed closures: if* $X \in \{\supseteq, f, f\supseteq\}$ *and* $\mathcal{T}, \mathcal{T}' \subseteq LGO_N$, *then* $\mathcal{T} \subseteq \overline{\mathcal{T}}^X$ *and* $\overline{\overline{\mathcal{T}}^X}^X = \overline{\mathcal{T}}^X$; *also, if* $\mathcal{T} \subseteq \mathcal{T}'$ *then* $\overline{\mathcal{T}}^X \subseteq \overline{\mathcal{T}'}^X$.

Proof. The first and third conditions are immediate from the definitions. As for the second, let $X_0 a_1 X_1 \ldots a_n X_n \in \overline{\overline{\mathcal{T}}^f}^f$. Then, there exists $Y_0 a_1 Y_1 \ldots a_n X_n \in \overline{\mathcal{T}}^f$ and thus there exists $Z_0 a_1 Z_1 \ldots a_n X_n \in \mathcal{T}$, which implies $X_0 a_1 X_1 \ldots a_n X_n \in \overline{\mathcal{T}}^f$; the inclusion in the other direction follows from monotonicity. Analogously for the other two operators. □

Proposition 10. *For all* $X \in \{\supseteq, f, f\supseteq\}$, $\mathcal{T} \leq_N^{lX} \mathcal{T}'$ *iff* $\overline{\mathcal{T}}^X \subseteq \overline{\mathcal{T}'}^X$.

Proof. It is easy but tedious, so only the case $X = f\supseteq$ is presented in detail. Assume $\mathcal{T} \leq_N^{lf\supseteq} \mathcal{T}'$: for all $t = X_0 a_1 X_1 \ldots a_n X_n \in \mathcal{T}$ there exists $Y_0 a_1 Y_1 \ldots a_n Y_n \in \mathcal{T}'$ with $X_n \supseteq Y_n$ and hence $t \in \overline{\mathcal{T}'}^{f\supseteq}$ and $\mathcal{T} \subseteq \overline{\mathcal{T}'}^{f\supseteq}$; $\overline{\mathcal{T}}^{f\supseteq} \subseteq \overline{\mathcal{T}'}^{f\supseteq}$ follows because of the properties of closures.

Conversely, from $\overline{\mathcal{T}}^{f\supseteq} \subseteq \overline{\mathcal{T}'}^{f\supseteq}$ it follows that $\mathcal{T} \subseteq \overline{\mathcal{T}'}^{f\supseteq}$ and, thus, for all $X_0 a_1 X_1 \ldots a_n X_n \in \mathcal{T}$ there exists $Y_0 a_1 Y_1 \ldots a_n Y_n \in \mathcal{T}'$ with $X_n \supseteq Y_n$: therefore $\mathcal{T} \leq_N^{lf\supseteq} \mathcal{T}'$. □

Let us see which of the semantics in the spectrum are characterized by the orders $\leq_N^{lX}$ defined above.

Proposition 11. *For $N = U$ we have $\leq_U^l = \leq_U^{l\supseteq} = \leq_U^{lf} = \leq_U^{lf\supseteq} = \sqsubseteq_T$. As a consequence, the only semantics coarser than plain simulation that can be characterized by means of linear observations using L_U is the trace semantics.*

Proof. The first three equalities are obvious since U provides useless (empty) local information ($L_U = \{\cdot\}$). The last equality was proved in Proposition 6(1). □

Proposition 12. *For $N = C$ we have $\leq_C^l = \leq_C^{l\supseteq} = \leq_C^{lf} = \leq_C^{lf\supseteq} = \sqsubseteq_{CT}$. As a consequence, the only semantics coarser than complete simulation that can be characterized by means of linear observations using L_C is the complete trace semantics.*

Proof. Note that the local information at the intermediate steps of traces in LGO_C has to be *false*, since it corresponds to non-terminated states; thus, only the final states provide real information. Since in this case $\supseteq$ corresponds to Boolean equality, the first three equalities follow; the fourth was proved in Proposition 6(3). □

Proposition 13. *For $N = I$, $\leq_I^{lf\supseteq}$ characterizes the failures semantics, $\leq_I^{lf}$ the readiness semantics, $\leq_I^{l\supseteq}$ the failure traces semantics, and $\leq_I^l$ the ready trace semantics. Therefore, the possible worlds semantics is the only semantics in the ltbt spectrum coarser than ready simulation that cannot be characterized using lgo_I's.*

Proof. We have already proved (Propositions 6 and 8) the four characterizations, while $\sqsubseteq_{PW}$ cannot be characterized using lgo_I's because all the information available in our lgo_I's was needed to capture the ready trace semantic and it is well-known that the possible world semantics is strictly finer. □

As we will see in Section 5, the possible world semantics is the only deterministic branching semantics in the spectrum and will require the use of the deterministic branching observations introduced there to be characterized in an observational way.

Proposition 14. *1. $\leq_T^{lf}$ is the possible futures preorder;*
2. $\leq_T^{lf\supseteq}$ is the impossible futures preorder.

Proof. 1. Obvious.
2. Assume that $p \leq_T^{lf\supseteq} q$. Then, $p \stackrel{\alpha}{\Longrightarrow} p'$, with $\alpha = a_1 \dots a_n$ implies $q \stackrel{\alpha}{\Longrightarrow} q'$ with $T(q') \subseteq T(p')$. Therefore, if $p \stackrel{\alpha}{\Longrightarrow} p'$ with $T(p') \cap X = \emptyset$, then $q \stackrel{\alpha}{\Longrightarrow} q'$ with $T(q') \cap X = \emptyset$ which implies $p \sqsubseteq_{IF} p'$.
   Conversely, if $p \sqsubseteq_{IF} q$ and $t = X_0 a_0 X_1 \dots X_n \in LGO(p)$ then, if $p \stackrel{\alpha}{\Longrightarrow} p'$ with $\alpha = a_1 \dots a_n$, $T(p') \cap T(p')^c = \emptyset$, which implies $q \stackrel{\alpha}{\Longrightarrow} q'$, $T(q') \cap T(p')^c = \emptyset$. Hence, there exists $t' = X_0' a_0 X_1' \dots X_n' \in LGO(q)$ with $T(q') \subseteq T(p')$, which implies $p \leq_T^{lf\supseteq} q$. □

As a matter of fact, the possible futures semantics is just below the 2-nested simulation semantics in the spectrum, only because the trace simulation semantics is missing there. The impossible futures semantics has been introduced quite recently [8] and is not yet well-known. And it is at this point where we have discovered our first two "lost creatures", defined as follows.

Definition 12. *The* possible futures trace semantics *is defined by* lgo_T*'s related by* $\leq^l_T$ *and the* impossible futures trace semantics *is defined by* $\leq^{l\supseteq}_T$.

Let us complete this part of the new extended spectrum by introducing the diamond generated by lgo_S's. This gives us four new semantics coarser than 2-nested semantics. For instance, for the case of failures we obtain the following definition.

Definition 13. *The* extended simulation failures *of a process* p *are defined as* $ExtSimFailures(p) = \{\langle\alpha, p''\rangle \mid \alpha \in A^*, p \stackrel{\alpha}{\Longrightarrow} p', p' \sqsubseteq_S p''\}$. *The* simulation failures *of a process* p *are defined as* $SimFailures(p) = \{\langle\alpha, B\rangle \mid p \stackrel{\alpha}{\Longrightarrow} p', B \cap BGO_U(p') = \emptyset\}$. *We write* $p \sqsubseteq_{SF} q$ *iff* $SimFailures(p) \subseteq SimFailures(q)$.

It can be proved that the inclusion $SimFailures(p) \subseteq SimFailures(q)$ holds if and only if $ExtSimFailures(p) \subseteq ExtSimFailures(q)$. Thus, simulation failures are essentially defined by translating the characterization of ordinary failures with the closure of readiness.

Proposition 15. $\leq^{lf\supseteq}_S = \sqsubseteq_{SF}$.

Proof. Analogous to the characterization of $\sqsubseteq_F$ in terms of $\leq^{lf\supseteq}_I$. □

5 Deterministic branching observations

Definition 14. *1. We say that a bgo is* deterministic *if for every node in it, its set of children* $\{(a_i, bgo_i)\}$ *satisfies* $a_i \neq a_j$ *whenever* $i \neq j$. *We denote with* $dBGO_N$ *the set of deterministic observations in* BGO_N.

2. The set of deteministic branching observations (dbgo for short) of a process p *is* $dBGO_N(p) = BGO_N(p) \cap dBGO_N$.

3. We write $p \leq^{db}_N q$ *if* $dBGO_N(p) \subseteq dBGO_N(q)$.

Like the linear observations, the set $dBGO_N(p)$ can be defined recursively and the corresponding semantics, compositionally.

Example 2. For the two processes $p = a(bc + bd)$ and $q = abc + abd$ we have that both deterministic observations in Figure 5 belong to $dBGO_I(p)$ and $dBGO_I(q)$. Indeed, that must be the case since it is easy to check that $dBGO_I(p) = dBGO_I(q)$.

In order to prove that dbgo's for the constraint I characterize the possible world semantics we first recall the definition of that semantics in [7].

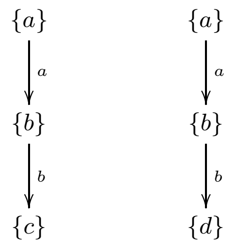

Fig. 5. Deterministic branching observations

Definition 15. *A deterministic process p is a possible world of a process q if $p \sqsubseteq_{RS} q$. The set of possible worlds of p is denoted by $PW(p)$. We define the order $p \sqsubseteq_{PW} q$ iff $PW(p) \subseteq PW(q)$.*

When defining the possible worlds of a process we have to solve all the nondeterministic choices in it, each choice leading to one of its possible worlds. The same idea supports the selection of dbgo's to characterize this semantics: the nondeterministic branching observations in $BGO_N(p)$ are not present in $dBGO_N(p)$, where we have instead all the possible deterministic subtrees of every branching observation.

In our proof below we will relate the dbgo's in $dBGO_I(p)$ and the possible worlds in $PW(p)$. When necessary, we will consider observations in $dBGO_I(p)$ as processes in BCCSP by removing the information from their nodes; by abuse of notation we will also denote with *dbgo* the process obtained after such a removal. Also, we call *complete* those observations that, for every node labeled by an offering A, have a branch labeled by each of the actions in A.

Definition 16. *The set of complete deterministic branching observations for the local observation function L_I is the set $cdBGO_I \subseteq dBGO_I$ recursively defined as:*

- $\langle \emptyset, \emptyset \rangle \in cdBGO_I$.
- $\langle A, \{(a, cdbgo_a) \mid a \in A\} \rangle \in cdBGO_I$ *for every* $a \in A$ *and* $cdbgo_a \in cdBGO_I$.

For each $p \in BCCSP$ we define its set of complete deterministic branching observations $cdBGO_I(p) = dBGO_I(p) \cap cdBGO_I$.

We also associate to a deterministic process q its universal (complete deterministic) branching observation.

Definition 17. *For a deterministic process p, its universal deterministic branching observation $cdbgo(p)$ is:*

- $cdbgo(\mathbf{0}) = \langle \emptyset, \emptyset \rangle$.
- $cdbgo(\sum_{a \in A} ap_a) = \langle A, \{(a, cdbgo(p_a)) \mid a \in A\} \rangle$.

The following result is then immediate.

Proposition 16. *For every $p \in BCCSP$, $cdbgo(p) \in cdBGO_I(p)$.*

Lemma 1. *For every $q \in PW(p)$, $cdbgo(q) \in cdBGO_I(p)$.*

Proof. By structural induction on q:

- If q is $\mathbf{0}$, then $p \equiv \mathbf{0}$ and $\langle \emptyset, \emptyset \rangle \in cdBGO_I(\mathbf{0})$.
- If q is $\sum aq_a$, since $q \in PW(p)$ we have $q \sqsubseteq_{RS} p$. This implies $I(q) = I(p)$ and that, for all $a \in A$, there exists $p \xrightarrow{a} p_a$, $q_a \sqsubseteq_{RS} p_a$, so that $q_a \in PW(p_a)$. By induction hypothesis, $cdbgo(q_a) \in cdBGO_I(p)$. Now, by definition, $cdbgo(q) = \langle A, \{(a, cdbgo(q_a)) \mid a \in A)\} \rangle$ and, from $p \xrightarrow{a} p_a$ and $I(p) = I(q)$, we conclude $cdbgo(q) \in dBGO_I(p)$ and therefore $cdbgo(q) \in cdBGO_I(p)$. $\square$

Lemma 2. *For every process q such that $cdbgo(q) \in cdBGO_I(p)$ we have $q \sqsubseteq_{RS} p$ and therefore $q \in PW(p)$.*

Proof. We will prove that the set $S = \{(q,p) \mid cdbgo(q) \in cdBGO_I(p)\}$ is a ready simulation. Obviously, for $(q,p) \in S$ it is $I(q) = I(p)$ and, if $q \xrightarrow{a} q_a$, there exists $p \xrightarrow{a} p_a$ with $cdbgo(q_a) \in cdBGO_I(p_a)$, which shows that $(q_a, p_a) \in S$ and that S is a ready simulation. $\square$

Theorem 3. *For all processes $p_1, p_2 \in BCCSP$, $p_1 \sqsubseteq_{PW} p_2$ iff $p_1 \leq_I^{db} p_2$.*

Proof. ($\Leftarrow$) For $q \in PW(p_1)$, by Lemma 1 we have $cdbgo(q) \in cdBGO_I(p_1)$ and therefore $cdbgo(q) \in cdBGO_I(p_2)$. Now, by Lemma 2, $q \sqsubseteq_{RS} p_2$ and thus $q \in PW(p_2)$.

($\Rightarrow$) Let $dbgo \in dBGO_I(p_1)$: by definition of $dBGO_I(p_1)$ it is clear that we can extend $dbgo$ into some $dbgo' \in cdBGO_I(p_1)$. Now, by Lemma 2, $dbgo' \sqsubseteq_{RS} p_1$ (taking $dbgo'$ as a deterministic process). Therefore, $dbgo' \in PW(p_1)$ and thus $dbgo' \in PW(p_2)$ and, by Lemma 1, $cdbgo(dbgo') = dbgo' \in cdBGO_I(p_2)$: hence $dbgo \in dBGO_I(p_2)$ as required. $\square$

Let us briefly consider the remaining new semantics definable by means of deterministic branching observations. It is clear that in all cases the corresponding orders verify $\leq_N^b \subseteq \leq_N^{db} \subseteq \leq_N^l$, so that the associated semantics will be situated between the corresponding semantics defined by branching observations in BGO_N and linear observations in LGO_N, as is the case for the possible worlds semantics, located between the ready simulation semantics and the ready trace semantics.

Admittedly, most of these semantics are rather strange and this is probably the reason why, as far as we know, they have not been previously considered. However, the simplest of them all, that corresponding to $N = U$, has properties similar to the possible worlds semantics and, in fact, can be defined by simply removing from its definition the "R" in the condition $q \sqsubseteq_{RS} p$. Hence, we can regard as possible worlds those deterministic implementations where we offer just a part of the action offered by the given process.

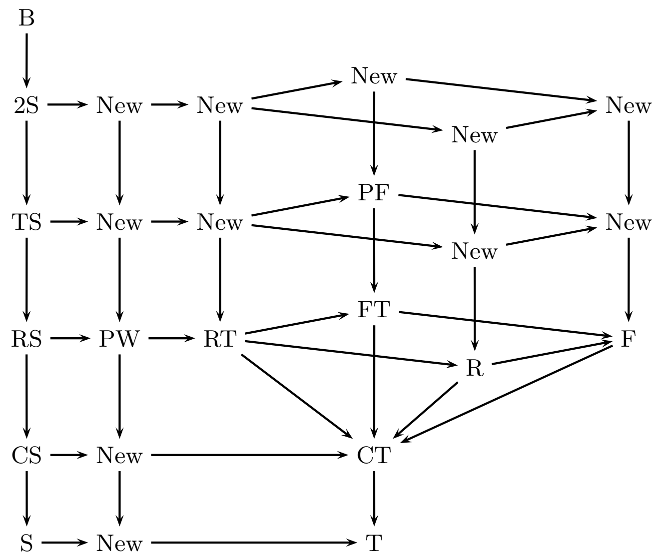

Fig. 6. The new linear time-branching time spectrum

Definition 18. *The partial possible worlds of a process p are those deterministic processes that verify $q \sqsubseteq_S p$. We denote with $PW_U(p)$ the set of partial possible worlds of a process p and define $p \sqsubseteq_{UPW} q$ if $PW_U(p) \subseteq PW_U(q)$.*

Proposition 17. *For all processes $p_1, p_2 \in BCCSP$, $p_1 \sqsubseteq_{UPW} p_2$ iff $p_1 \leq_U^{db} p_2$.*

Proof. Similar to Theorem 3, simplified by the fact that all *dbgo* in $PW_U(p)$ satisfy $dbgo \sqsubseteq_S p$. □

Example 3. We have $a \sqsubseteq_{UPW} a + b$ since $\langle \cdot, \{(a, \emptyset)\} \rangle \in dBGO_U(a + b)$. By contrast, for $p = ab + ac$ and $q = a(b + c)$ we have $p \sqsubseteq_{UPW} q$ but $q \not\sqsubseteq_{UPW} p$ because $\langle \cdot, \{(a, \langle \cdot, \{(b, \langle \cdot, \emptyset \rangle), (c, \langle \cdot, \emptyset \rangle)\} \rangle)\} \rangle \in dBGO_U(q) - dBGO_U(p)$.

Analogously, for any other constraint N we could define the N-possible worlds using $\sqsubseteq_{NS}$, which in turn would be characterized using the observations in $dBGO_N$.

The extended spectrum can now be depicted as in Figures 6 and 7.

6 Back to branching observations

For each of the three orders we defined over linear observations to characterize some semantics in Section 4, the local information we were interested in was restricted. The same scheme can be generalized to the branching observations. This way, for each constraint N we would obtain three new branching semantics

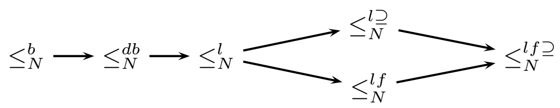

Fig. 7. Basic slice in the linear time-branching time spectrum

based on bgo's in BGO_N which, together with the original N-simulation semantics, would constitute a diamond of branching semantics at a higher level in our extended ltbt-spectrum. The introduction of these new semantics also offers a more clear view of the spectrum, with two main levels of branching and linear semantics and an intermediate one of deterministic branching semantics. Though this provides the means for obtaining a host of new semantics, it is also true that most of them are bizarre, in sharp contrast with the fact that the corresponding orders gave rise to interesting semantics when applied to linear observations.

To illustrate the comments above, next we consider in some detail the case $N = I$, which corresponds to the most interesting semantics.

Definition 19. *For $bgo, bgo' \in BGO_I$ we define:*

– $bgo \leq_I^{\supseteq} bgo' \iff \big(bgo = \langle A_1, S_1\rangle$ *and* $bgo' = \langle A_2, S_2\rangle$ *and* $A_1 \supseteq A_2$ *and* $S_1 = \{(a_i, bgo_i) \mid i \in I\}$ *and* $S_2 = \{(a_i, bgo'_i) \mid i \in I\}$ *and for all* $i \in I$ $(bgo_i \leq_I^{\supseteq} bgo'_i)\big)$.

– $bgo \leq_I^{f} bgo' \iff \big(bgo = \langle A_1, \emptyset\rangle$ *and* $bgo' = \langle A_1, \emptyset\rangle\big)$ *or* $\big(bgo = \langle A_1, S_1\rangle$ *and* $bgo' = \langle A_2, S_2\rangle$ *and* $S_1 = \{(a_i, bgo_i) \mid i \in I\}$ *and* $S_2 = \{(a_i, bgo'_i) \mid i \in I\}$ *and for all* $i \in I$ $(bgo_i \leq_I^{f} bgo'_i)\big)$.

– $bgo \leq_I^{f\supseteq} bgo' \iff \big(bgo = \langle A_1, \emptyset\rangle$ *and* $bgo' = \langle A_2, \emptyset\rangle$ *and* $A_1 \supseteq A_2\big)$ *or* $\big(bgo = \langle A_1, S_1\rangle$ *and* $bgo' = \langle A_2, S_2\rangle$ *and* $S_1 = \{(a_i, bgo_i) \mid i \in I\}$ *and* $S_2 = \{(a_i, bgo'_i) \mid i \in I\}$ *and for all* $i \in I$ $(bgo_i \leq_I^{f\supseteq} bgo'_i)\big)$.

Definition 20. *For $\mathcal{B}, \mathcal{B}' \subseteq BGO_I$ and $X \in \{\supseteq, f, f\supseteq\}$, we define the orders $\leq_I^{bX}$ by:*

– $\mathcal{B} \leq_I^{bX} \mathcal{B}' \iff$ *for all* $bgo \in \mathcal{B}$ *there exists* $bgo' \in \mathcal{B}'$ *with* $bgo \leq_I^X bgo'$.

Then, we write $p \leq_I^{bX} q$ if $BGO_I(p) \leq_I^{bX} BGO_I(q)$.

It is somewhat surprising to discover that $\leq_I^{b\supseteq} = \leq_I^b$, since this was not the case for their linear "projections" $\leq_I^{l\supseteq}$ and $\leq_I^l$.

Proposition 18. *For all processes $p_1, p_2 \in BCCSP$, $p_1 \leq_I^{b\supseteq} p_2$ iff $p_1 \leq_I^b p_2$.*

Proof. Assume that $p_1 \leq_I^{b\supseteq} p_2$ and let $bgo \in BGO_I(p_1)$: it is clear that it can be extended into a complete $cbgo \in BGO_I(p_1)$. Then, there exists some $cbgo' \in BGO_I(p_2)$ with $cbgo \leq_I^{\supseteq} cbgo'$ and, since $cbgo$ is complete, $cbgo = cbgo'$ and hence $bgo \in BGO_I(p_2)$. The other implication is trivial. □

Example 4. For $p_1 = a(b+c)$ and $p_2 = ab+ac$, $p_1 \leq_I^{l\supseteq} p_2$ but $p_1 \not\leq_I^{l} p_2$. However, $p_1 \not\leq_I^{b\supseteq} p_2$ since for $bgo = \langle\{a\}, (a, \langle\{b, c\}, \{(b, \emptyset), (c, \emptyset)\}\rangle)\rangle \in BGO_I(p_1)$ there is no $bgo' \in BGO_I(p_2)$ with $bgo \leq_I^{l\supseteq} bgo'$.

By contrast, the branching semantics defined by $\leq_I^{bf}$ and $\leq_I^{bf\supseteq}$ are indeed new.

Example 5. For the processes p and q in Figure 8, $p \leq_I^{bf} q$ but $p \not\leq_I^{b} q$.

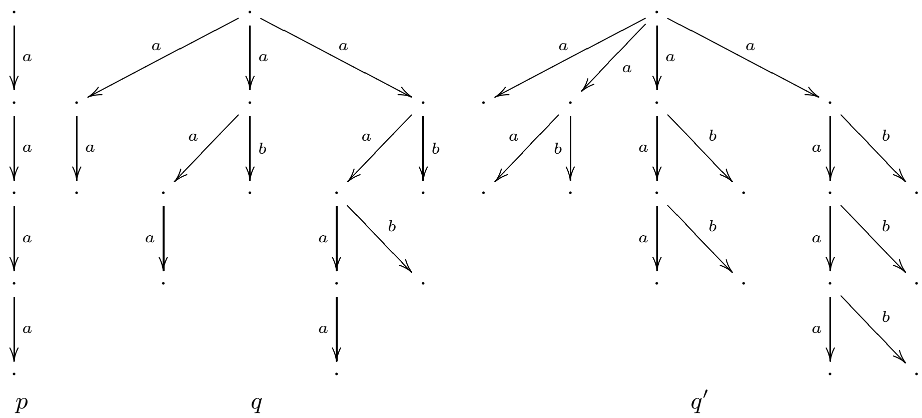

Fig. 8. Three processes

This example shows that it is quite difficult to characterize this semantics as a simulation-like one. Furthermore, we conjecture that it is not finitely axiomatizable. As a matter of fact, we have found an involved characterization as a simulation semantics which combines plain and ready simulation using the branching observations of the compared processes.

Definition 21. *We say that $R \subseteq BGO_I \times BCCSP$ is a* final-ready simulation *when:*

- *$(\langle A, \emptyset\rangle, q) \in R$ implies $I(q) = A$.*
- *$(\langle A, \{(a_i, bgo_i)\}\rangle, q) \in R$ implies that for all $i \in I$ there exists $q \xrightarrow{a_i} q_i$ such that $(bgo_i, q_i) \in R$.*

We say that p is final-ready simulated by q when for all bgo $\in BGO_I(p)$ *there exists a final-ready simulation R with* $(bgo, q) \in R$*, and write* $p \sqsubseteq_{fRS} q$.

Theorem 4. *For all* $p, q \in BCCSP$*,* $p \sqsubseteq_{fRS} q$ *iff* $p \leq_I^{bf} q$.

Example 6. It is easy to check that for p and q' as in Figure 8 we have $p \leq_I^{bf\supseteq} q'$ but $p \not\leq_I^{bf} q'$.

Final failure simulations are defined exactly like final-ready simulations but substituting $I(q) \subseteq Act$ for $I(q) = A$ in the first clause. Then the order $\sqsubset_{fFS}$ between processes is defined.

Theorem 5. *For all* $p, q \in BCCSP$*,* $p \sqsubseteq_{fFS} q$ *iff* $p \leq_I^{bf\supseteq} q$.

As previously noted, these are certainly bizarre semantics but we believe it interesting to indicate their existence because their definitions in terms of branching observations look, by analogy to the linear case, quite natural. However, it also seems that when dealing with branching observations the introduction of any kind of asymmetry in the treatment of local observations produces quite involved semantics.

7 Conclusion

In this paper we have presented the first part of our unification work on the semantics for concurrency. We have seen that the branching-linear character of a semantics is the main fact to take into account in order to classify it properly. Indeed, this is not very surprising since the spectrum of semantics of concurrency was already called the linear time-branching time spectrum. The important result of our work is that all the branching semantics can be observationally characterized in a uniform way, so that the only difference between them is the local observation function L_N used to watch the states of the processes. We have also uncovered the common structure of the diamonds under each simulation semantics, that corresponds to different orders on the sets of linear observations and are defined in the same way for all the constraints N. Finally, we have found a single semantics in the ltbt spectrum that is defined by deterministic branching observations.

We think that this unification work sheds light on the structure of the spectrum. Besides, and more importantly, with the uniform descriptions of the semantics it will be much easier to prove general properties satisfied by all of them by means of parameterized proofs for generic constraints N, and also by considering the four orders defining the linear semantics corresponding to each constraint in a homogeneous way. In fact, as it was already mentioned in the introduction, we have found a common framework in which the axiomatizations of every semantics are particular cases of a couple of parametrized axioms; by using the uniform characterizations as definitions of the semantics we have already proved the soundness and completeness of the new axiomatizations for the semantics without having to resort to cumbersome proofs by cases.

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A A preview of the second part of the unification theory

We believe our unification work at the level of observational semantics to have been a success by identifying the three families of semantics respectively characterized by branching, linear, and deterministic branching observations, as well as the four orders for the linear semantics coarser than a given constrained simulation. In addition, these results have an axiomatic counterpart where the unification is even more clearly displayed.

We have discovered two basic axioms that characterize the branching semantics and the linear semantics in the diamonds under any constrained simulation semantics:

- $(S)\quad x \preceq x + y$.
- $(F)\quad a(x+y) \preceq ax + a(y+w)$.

In order to obtain the axiomatization of each constrained N-simulation semantics we turn (S) into a conditional axiom that takes into account the corresponding constraint N:

$$(NS)\quad N(x,y) \Rightarrow x \preceq x + y\,.$$

With respect to the linear semantics coarser than an N-simulation, we have to add to (NS) the adequate conditional version of (F), which by itself would axiomatize the finest semantics in each of the diamonds. Thus we obtain

$$(RND) \quad M(x,y,w) \Rightarrow a(x+y) \preceq ax + a(y+w)\,.$$

In order to illustrate the conditions that define each of the semantics in the diamonds, let us show those for the semantics in the diamond below ready simulation ($N = I$):

- $M_F(x,y,w) ::= \text{BCCSP} \times \text{BCCSP} \times \text{BCCSP}$.
- $M_R(x,y,w) \iff I(x) \supseteq I(y)$.
- $M_{FT}(x,y,w) \iff I(w) \subseteq I(y)$.
- $M_{RT}(x,y,w) \iff I(x) = I(y)$ and $I(w) \subseteq I(y)$.

In the general case, instead of RND and subset inclusion we should consider its equational version and the preorder $\leq_N$ so that:

$$(RND_{\equiv}) \quad M(x,y,w) \Rightarrow ax + a(y+w) + a(x+y) = ax + a(y+w)$$

and

- $M_F(x,y,w) ::= \text{BCCSP} \times \text{BCCSP} \times \text{BCCSP}$.
- $M_R(x,y,w) \iff [\![y]\!]_N \leq_N [\![x]\!]_N$.
- $M_{FT}(x,y,w) \iff [\![w]\!]_N \leq_N [\![y]\!]_N$.
- $M_{RT}(x,y,w) \iff ([\![x]\!]_N = [\![y]\!]_N)$ and $([\![w]\!]_N \leq_N [\![y]\!]_N)$.

We have proved that these axioms provide a complete axiomatization of all the linear semantics corresponding to $N \in \{U, C, I, T\}$ and, in particular, for all the linear semantics in the classic ltbt-spectrum. We have that $\{B_1\text{–}B_4, (NS)\}$ is an axiomatization of N-constrained simulations, where B_1–B_4 are the well-known axioms for bisimulation semantics, and $\{B_1\text{–}B_4, (NS), (RND^N_{\equiv,X})\}$ is a complete axiomatization of the four linear semantics in the diamond under N-simulation semantics. In particular, the possible worlds semantics is axiomatized by $\{B_1\text{–}B_4, (TS), (RND^T_{\equiv,R})\}$, where

$$\begin{array}{ll} (TS) & T(x) = T(y) \Rightarrow x \preceq x + y \\ (RND^T_{\equiv,R}) & T(y) \subseteq T(x) \Rightarrow ax + a(y+w) + a(x+y) = ax + a(y+w) \end{array}$$

Finally, for the deterministic branching semantics we only have to add to the corresponding axiom (NS) the possible worlds axiom

$$(PW) \quad a(bx + by + z) = a(bx + z) + a(by + z)$$

that, from left to right, states that any non-deterministic process can be split into its deterministic implementations. For the possible worlds semantics the axiomatization $\{B_1\text{–}B_4, (S), (PW)\}$ is obtained.

We also have simple axiomatizations of the equivalences induced by the semantics, obtained by replacing the N-simulation axiom (NS) by the corresponding equivalence axiom $(NS_{\equiv})$ $N(x,y) \Rightarrow a(x+y) = a(x+y) + ax$. This result has been already proved in [4], where an example of a generic proof valid for any semantics fulfilling some natural conditions can be found.